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The analytical predictions on displacement and stress around shallow tunnels subjected to surcharge loadings

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ABSTRACT

Tunnels in urban areas inevitably lie near the ground surface, and will be affected significantly by the loads on the surface. In this study, a new analytical solution is presented for the first time, by means of which the ground responses around circular shallow tunnels subjected to surcharge loadings can be quickly predicted. The ground is approximately assumed as an elastic or viscoelastic medium, and arbitrary distribution of surcharge loads, as well as any viscoelastic models, are accounted for.

Case (A) is modelled with surcharge loads applied before the tunnel excavation. The incremental elastic displacements and stresses which occur after the tunnel excavation are provided by Verruijt's method, combined with a Fourier expansion of the tractions. Case (B) represents the model in which the surcharge loads are applied after the tunnel excavation, and the elastic ground responses due to the presence of surcharge load are proposed by the superposition of modified Flamant's solutions and the solution of Case (A). The solutions for the viscoelastic problem of the two cases are also proposed by an extended corresponding principle, taking into account the time-dependence of the rock/soil with any types of the linear viscoelastic model.

All of these analytical solutions are verified by good agreements of the comparison between the analytical solutions and FEM results. Furthermore, they are consistent with measured data qualitatively. Finally, a parametric study is carried out to investigate the influence of the uniform surcharge load on the stresses and displacements of the ground in the two cases. The proposed new solutions potentially provide an alternative approach in the preliminary designs of future shallow tunnels.

1. Introduction

Tunnels in urban areas are usually constructed at shallow depths for the sake of low operational costs, and they inevitably lie in the region near the above-ground structures, or where foundation digging takes place. The extreme surcharge produced by the dumped soils from nearby construction site, as well as the effect of structures and the excavation of foundation pits, can be approximately equivalent to the loads imposed on ground surface (surcharge loads). According to the statistics of the number of recorded man-made tunnel accidents in Shanghai from 2004 to 2013 (Huang and Zhang, 2016), the accidents induced by extreme surcharge and nearby excavation are the most significant. On the other hand, tunnelling below ground surface with existing surcharge loadings will induce different ground movement and deformation, compared with that with stress free ground surface. The prediction of ground responses considering the surcharge loadings exerted on the surface before/after tunnel excavation is crucial in estimation of the stability and safety of shallow tunnels in excavation and operation period.

Though several numerical analyses have been presented in detailed analysis of shallow tunnelling with consideration given to more complex geological conditions (Liu et al., 2012; Prassetyo and Gutierrez, 2016; Jiang and Yin, 2014), as well as the interaction between tunnel and existing structures where structures and tunnels are both involved in the detailed modelling (Bilotta et al., 2017; Mirhabibi and Soroush, 2013; Yoo, 2013), they are found to be very expensive in terms of data preparation and computational time (Basile, 2014). Empirical formulas with simple expressions are widely used in engineering practice to predict the surface settlement under green-field situation, *e.g.* Gaussian equation suggested by Peck (1969) is one of the most widely-used empirical formulas based upon field observations. In order to improve the applicability of the formula to general cases, the Peck formula was then modified in other references (Osman et al., 2006a, 2006b). Empirical equations only valid under the condition of stress-free ground

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surface, and they are heavily subject to some important limitations. These include their applicability to different tunnel geometries, rock/soil properties and construction procedures.

As an alternative, the analytical approaches are important for understanding the mechanics of generation of stress and deformation, and allow the examination of the fundamental relationships that exist between the different variables and parameters involved in the problem to be made. In the present body of literature, no analytical solutions which consider the surcharge loads in shallow tunnelling are available. The aim in this study is to propose a new accurate solution for the predictions of ground responses, taken into account of surface loads applied before/after tunnel excavation.

Due to the presence of surface boundaries and surcharge loads, the analytical solutions of shallow tunnels are much more difficult to address than those of deep buried tunnels. Most of the studies available at present focused on the problem under the gravitational initial stress state. For instance, by introducing the bipolar coordinates, Jeffery (1921) and Mindlin (1940, 1948) provided the elastic analytical stresses induced by shallow tunnel excavation. More recently, Massinas and Sakellariou (2009) presented the analytical solution for the problems of the plastic zone and stress distribution around circular shallow tunnels in elasto-plastic half planes, which were derived using bipolar coordinates. However, the loads on the ground surface were not considered in these approaches, and it was difficult for these studies to provide the ground displacement fields.

Through the use of a singularity superposition technique, the stress-free ground surface could be considered by introducing the virtual sources in an infinite medium. Some results have been obtained for elastic half-spaces with incompressibility conditions (Sagaseta, 1988). Verruijt and Booker (1998) extended this method to more generalized conditions, such as the ovalization deformation of tunnel openings. Also, based on the complex formulation of planar elasticity and singularity superposition solutions, analytical solutions were presented by Zymnis et al. (2013) for ground deformations caused by shallow tunnelling in linear elastic soil masses with cross-anisotropic stiffness properties. However, although the singularity superposition technique provides a convenient method by which to access the displacement fields around shallow tunnels, it is an approximate approach with no consideration given to the tunnel boundary conditions. Therefore, it is only suited for cases which have stress-free surface boundary conditions.

A complex variable method (Muskhelishvili, 1966; Sokolnikoff and Specht, 1956) has been widely used to analyze the mathematical problems associated with underground construction, especially in the analyses of non-circular openings, and problems with multiple connected regions. Verruijt (1997, 1998) derived the analytical solutions by a complex variable method for a circular tunnel in an elastic half-plane, with the boundary conditions of a prescribed uniform radial displacement or radial stress. Then, by utilizing Verruijt's solutions (Verruijt and Booker, 1998), Wang et al. (2009) studied the ground deformations under four possible movement patterns of shallow tunnels, and carried out a comparison between the solutions and measured data. Fu et al. provided an analytical solution for shallow twin tunnels assuming an elastic half space (Fu et al., 2014), where only the gravitational initial stresses were considered. In all these studies, the variations with depth of the initial stresses due to gravity at the tunnel boundary were not considered. In order to address this limitation, Strack and Verruijt (2002) presented the solutions for a buoyant tunnel in an elastic half-plane by assuming two additional logarithmic terms in the potentials due to the resultant buoyancy force acting on the tunnel. Lu et al. (2016) recently provided the detailed analytical derivation and solutions for shallow tunnel excavations, and the linear variations of the initial stresses with depth were taken into account. Subaqueous tunnels are subjected to infinite uniform vertical load at the ground surface, which is different from the aforementioned studies regarding the stress-free conditions of the ground surface. The analytical solution of a subaqueous circular tunnel was proposed by Yu et al. (2014) by expanding the uniform surface load as a Fourier series.

In all the aforementioned research studies regarding ground deformations and stresses induced by tunnelling in gravitational half-plane, only stress-free or infinite uniform vertical loads at the ground surface were considered. However, due to the presence of buildings or loads on the ground surface in some cases, the equivalent surcharge loads should be taken into account. In this study, a new analytical solution will be derived for the first time to quickly determine the ground stresses and movements of two cases where the arbitrarily distributed surcharge loads are imposed on the ground surface before/after tunnel excavation. These solutions provide an alternative approach for the preliminary designs of future shallow tunnels.

2. Problem statement and mechanical models

2.1. Formulation of the problem

In this study, shallow tunnel excavations with loads imposed on the surface (surcharge loads) are considered. In the tunnel engineering, the following two models should be considered:

Case (A): The tunnel is excavated near existing structures. If the structural loads are simplified as surcharge loads on the ground surface, this case can be modelled as a tunnel which is being excavated after the application of surcharge loads on the surface.

Case (B): The structures or foundation pits are constructed near a shallow tunnel, or a surcharge is applied on the ground surface after tunnel excavation. The model of this case can be simplified as a tunnel subjected to loads on the surface with downward or upward directions after the tunnel excavation.

The stress and deformation of the ground of the aforementioned two models can be divided into two parts, as follows: the first is induced by the release of the initial stresses due to gravity, and the second is induced by the surcharge loads. Since the first part has been discussed or derived in the references by the empirical, analytical, or numerical methods (Do et al., 2014; Hasanpour et al., 2012; Mindlin, 1940; Peck, 1969; Verruijt, 1997), of which only the second parts, which are due to the surcharge loads, are analyzed in this study. The plane-strain conditions are considered, and the following assumptions are made:

(1) The circular tunnel with radius R_0 is excavated below the ground surface, with the buried depth being h;
(2) The geo-material is homogeneous, isotropic, and linearly elastic or viscoelastic;
(3) The surcharge loads are assumed to be the arbitrary distributed vertical loads q_j ($j = 1, \ldots, m$) exerted on the surface from point A_j to B_j.

The problem in the plane of the cross-section of the tunnel is shown in Fig. 1. Both the Cartesian coordinates (x,y) and polar coordinates (r,θ) will be employed in the derivation. The x coordinate of the midpoint of line A_jB_j, x_j ("load location" in short) is adopted to represent the location of the surcharge load. Also, the width of the load is represented by w_j.

2.2. Models in derivation

In Case (A), before the tunnel excavation, the half-plane medium without holes is subjected to surcharge loads (Model 1, with boundary stress shown in Fig. 2a), and the corresponding displacement, which is not significant in tunnel engineering, has occurred. The incremental displacements and stresses will then occur after the tunnel excavation,