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Counting errors as a window onto children's place-value concept

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1. Introduction

1.1. Place-value concept and mathematical learning

While numbers are arbitrary symbols, their formation relies on systematic, meaningful rules – namely the place-value concept. Such concept is essential when making sense of multi-digit numbers. Students should understand that each position in a number represents a power of ten, and that each digit in the number carries a place value depending on its position. Take "54" as example. The digit 4 in the ones place carries a place value of 4 (4×10^0); the digit 5 in the tens place carries a place value of 50 (5×10^1). Analogous to morphemes in alphabetical words, the place-value concept governs the organization of digits in a number – rendering it meaningful.

Having a good grasp of the place-value concept is crucial to mathematical learning (Chan, 2014; Chan, Au, & Tang, 2013, 2014; Chan & Ho, 2010; Wearne & Hiebert, 1994). Previous studies have shown that children's place-value understanding predicts their performance in comprehension and production of numbers (McCloskey, 1992), computation (e.g., addition and subtraction; Ho & Cheng, 1997), mathematical problem-solving (Collet, 2003; Dehaene & Cohen, 1997; Fuson, Wearne, et al., 1997), and early mathematical achievement (Miura & Okamoto, 1989). Children with subpar understanding of the place-value concept are prone to mathematical learning difficulties (Chan & Ho, 2010; Chan et al., 2014; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). With training in the place-value concept, children improve their arithmetic performance (Fuson, 1990; Fuson & Briars, 1990; Ho & Cheng, 1997; Jones, Thornton, & Putt, 1994). Hence the place-value concept plays a vital role in early mathematical learning.

1.2. Using base-ten blocks to illustrate the place-value concept

One traditional approach to illustrate the place-value concept is to use base-ten blocks. For one thing, young children lack the cognitive capacity to understand the logic behind the arbitrary place-value system (e.g., ten units in the ones place should be traded for one unit in the tens place; Chandler & Kamii, 2009; Fosnot & Dolk, 2001), so they need concrete materials to figure out how the digits in a number relate to each other (e.g., ten cubes are equivalent to one bar; Fuson, 1990; Fuson & Briars, 1990). For another thing, the sizes of the base-ten blocks being proportional to their quantities (e.g., ten cubes make one bar) helps children to relate the blocks with the base-ten grouping in the place-value numeration system (Hiebert & Carpenter, 1992). Indeed, the base-ten blocks have long been adopted as a useful tool to teach and assess children's place-value concept across cultures. Traditional textbooks show diagrams of base-ten blocks to illustrate how the model maps onto the digits in a number (e.g., "46" means four bars and six cubes). To tap into children's understanding of the place-value concept, children are asked to represent a multi-digit number using base-ten blocks (Miura, Okamoto, Kim, Steere, & Fayol, 1993; Naito & Miura, 2001; Saxton & Cakir, 2006) or translate a model of base-ten blocks into the corresponding number (Chan et al., 2014).

1.3. Children's counting strategies and the development of the place-value concept

Systematic examination of how children count the base-ten blocks can further provide useful insights into how they understand the place-value structure of multi-digit numbers. Based on their classroom observations, Fuson, Smith, and Lo Cicero (1997) described children's conceptual structures of two-digit numbers in relation to how they counted the base-ten blocks. These conceptual structures include unitary multi-digit, sequence-tens and ones, and separate-tens and ones. Consider the quantity fifty-four. Children who conceptualize a multi-digit number as an undi-

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vided entity (i.e., unitary multi-digit conception) would count from one: 1, 2, 3, 4 ... 54; children who see a number as a composition of groups of tens (i.e., sequence-tens and ones conception) would count by tens: 10, 20, 30, 40, 50, 51 ... 54; children who treat the digits in a number as units in different place holders (i.e., separate-tens and ones conception) would count the groups of tens and ones separately: 1, 2, 3, 4, 5 tens and 1, 2, 3, 4 ones.

Moreover, Chan et al. (2014) examined how Chinese kindergarteners and first graders counted base-ten blocks in non-canonical form – i.e., arrangement of blocks not mapping directly onto the base-ten numeration structure (e.g., "22" being represented by one bar and 12 cubes) – a task known as the Strategic Counting Task. They identified some major types of counting strategies – which mapped onto the conceptual structures of multi-digit numbers delineated by Fuson, Smith, et al. (1997) (i.e., counting-from-one strategy for unitary multi-digit conception; counting-by-tens strategy for sequence-tens and ones conception; counting-by-separating-ones-and-tens strategy for separate-tens and ones conception). Importantly, the study showed a developmental trend shifting from unitary multi-digit conception and sequence-tens and ones conception to separate-tens and ones conception. Such trend reflected how children's place-value understanding develops – from having no sense of partitioning to appreciation of the value represented by each constituent digit in a number.

Based on the previous findings, perhaps children's counting *errors* may inform where their problems with the place-value concept are. In general, academic achievement scores alone cannot tell us why children fail to learn, but children's error patterns can help by offering a window onto the underlying difficulties (Ashlock, 2010).

1.4. From counting errors to children's misconceptions

To our understanding, there has been no systematic study examining children's counting errors related to their place-value understanding. Previous studies examining children's counting errors usually asked children to count arrays of objects or watch a puppet count a series of items and then spot out any counting errors (e.g., Fuson, 1988; Marle, Chu, Li, & Geary, 2014). These studies focused particularly on children's understanding of the counting principles (Gelman & Gallistel, 1986). Few studies have identified counting errors related to the place-value concept. In verbal counting, many children are uncertain how to name the next number that bridges to a decade or hundred (e.g., 29, 39, 99) – which requires carrying over (Emerson & Babtie, 2015; Reys, Lindquist, Lambdin, & Smith, 2015). Some children make errors when counting in steps of powers of ten (Hansen, Drews, Dudgeon, Lawton, & Surtees, 2014). For example, when asked to count in steps of 1000 from 24,700, a child counts as 24,700, 35,700, 45,700, 557,000 and so on. One suggested reason is that the child is lacking of place-value concept and does not know which digit he should be increasing by 1 each time.

Very few studies have described children's errors when counting base-ten blocks. One exception is the work by Reys et al. (2015) – where children literally put together the numerals representing the face values of sub-groups of blocks to form a total number. For example, when shown a sheet representing *one hundred* squares on the left and a group of *three* squares on the right (thus making the number "103"), a child assigns the numeral "1" to the sheet and the numeral "3" to the group of squares and then literally put the two numerals together to arrive at "13" as the total number. The authors suggest that such error may reflect poor understanding of zero as a place-holder. Although it seems that children's counting errors may provide important hints of their place-value problems, we need a systematic examination of these errors in order to gain a fuller picture.

1.5. Present study

To our knowledge, this study is the first to elicit and analyze children's errors when counting the traditional base-ten blocks to understand their place-value concept. In particular, we ask two important questions: (1) how do children's errors in counting the traditional base-ten blocks change in the first elementary school year? (2) is there any early sign of *specific* difficulties in place-value understanding for children who turn out to be weak in mathematics? To address these questions, we compared three groups of children – who were low-achieving, average-achieving, and high-achieving in mathematics at the end of first grade. We examined their errors in counting the traditional base-ten blocks twice – at the end of first and second semesters of first grade – to reveal how children's counting errors changed over the first elementary school year and to see if children who turned out to perform differently in mathematics by the end of first grade had already shown different patterns of counting errors at some points of time during their first year of study – which would then suggest that children who stuck in specific aspects of place-value concept (thus committing certain types of errors) during their first year of study would end up lagging behind their peers in mathematics. This would help us to gain better understanding of where children's early difficulties in the place-value concept are and what we can do to address their difficulties in daily instruction.

2. Method

2.1. Participants

Four hundred and thirty-three Chinese-speaking Grade 1 children (mean age = 6 years 8 months, 242 boys and 191 girls), recruited randomly from 21 primary schools in Hong Kong through a large-scaled longitudinal study, participated with written parental consent. These primary schools were government schools or aided schools run by local charitable and religious organizations, where students did not need to pay tuition fee. All children had normal intelligence and had never been diagnosed with dyslexia. Based on the mathematics achievement test at the end of the second semester, children who scored below the 20th percentile were classified as low-achieving, whereas those who scored between the 40th and 60th percentile were classified as average-achieving and those who scored at or above the 80th percentile were classified as high-achieving. As a result, there were 87 (48 boys and 39 girls) low-achieving children, 102 (50 boys and 52 girls) average-achieving children, and 86 (51 boys and 35 girls) high-achieving children. For the purpose of this study, we would report the data based on these three groups of children.

2.2. Materials and procedure

There were two phases – at the end of the first and second semesters of the first grade. In the first phase, children were asked to complete a counting task. In the second phase, they completed the same counting task again, as well as a mathematics achievement test. All the tasks were administered by trained researchers at school.

2.2.1. Strategic counting task

We adopted the Strategy Counting Task developed by Chan et al. (2014). In the familiarization phase, a child manipulated magnets of small squares (1.5 cm × 1.5 cm), bars (1.5 cm × 15 cm), and big squares (15 cm × 15 cm). The child was asked to line up 10 small squares and note their equivalence to one bar; likewise for 10 bars and one big square. In the testing phase, the child saw