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Looking back to the roots of partially correct constructs: The case of the area model in probability

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ABSTRACT

We use the notion Partially Correct Constructs (PaCCs) for students' constructs that partially match the mathematical principles underlying the learning context. A frequent expression of partial construction of mathematical principles is that a student's words or actions provide an inaccurate or misleading picture of the student's knowledge. In this study, we analyze the learning process of a grade 8 student, who learns a topic in elementary probability. The student successfully accomplishes a sequence of several tasks without apparent difficulty. When working on a further task, which seems to require nothing beyond his proven competencies, he encounters difficulties. Using the epistemic actions of the RBC model for abstraction in context as tracers, we analyze his knowledge constructing processes while working on the previous tasks, and identify some of his constructs as PaCCs that are concealed in these processes and explain his later difficulties. In addition, our research points to the complexity of the knowledge structures students are expected to deal with in their attempts to learn an elementary mathematical topic with understanding.

1. Introduction

The match between the mathematical principles underlying the learning context and students' constructs for these principles is, in many cases, only partial, even if these principles are adapted to the students and their learning processes, as is the case of the present research. And while extensive research has been devoted separately to knowledge construction and to alternative conceptions, there is a need to combine these research directions in order to better understand the processes by which knowledge is constructed, whether fully or only partially correct.

In a previous paper (Ron, Dreyfus & Hershkowitz, 2010), we proposed the notion of Partially Correct Construct (PaCC) for a student's knowledge construct that only partially matches a mathematical principle that underlies the learning context; we presented a view of processes of knowledge construction leading to partially correct constructs, as a tool for interpreting situations in which a student's answers to different parts of the same question seem to contradict each other. In the present paper, we further establish and elaborate the notion of PaCC and consider the development of PaCCs over several lessons, their persistence and consolidation or, alternatively, recession and disappearance.

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The research presented here is part of a larger project dealing with the construction and consolidation of knowledge in probability. From the large body of data that the project has produced we focus in the present paper on a learning sequence that consists of two and a half lessons of 45 min each, which is based on a set of tasks that differ from each other less in their mathematical structure than in their problem situations and in the amount of guidance given to the students. The learning environment included whole class discussions, small group learning, and homework. It enabled students to construct their individual solution strategies and self-explanations. In this learning sequence we looked for episodes where a student's later work is likely to be unexpected for the teacher, in view of this student's earlier performance, given that in a regular classroom a teacher does not usually have the opportunity to analyze her students' learning processes in detail.

The following case, which is typical for many others, exemplifies such an episode: Roni, an 8th grade student, learned to calculate probabilities in binomial two-dimensional sample space by means of an area model. He successfully accomplished a sequence of several tasks without expressing any difficulty. When working on a specific further task, which seemed to require only actions that he had not only successfully carried out, but also substantiated with correct and compelling explanations in earlier tasks, he ran into difficulties. In the present paper, we analyze Roni's work along the learning sequence that ends with the specific task; we use Roni's PaCCs to explain his difficulties, and show that these PaCCs are concealed in his knowledge constructing processes while working on the previous tasks. Moreover, we present findings from other students' work on the same task sequence. These findings shed more light, not only on Roni's knowledge construction but also as on how the analysis of students' work impacts the choice and refinement of the knowledge elements that compose the mathematical principles underlying the learning context. In this sense, the findings of our research point to the complexity of the knowledge students need in order to use the area model with understanding.

The paper is structured as follows: We begin with relevant research on learning probability, as well as a brief review of the RBC model and our previous work on PaCCs (Section 2), followed by a description of the research setting (Section 3). The key episode that motivated us to choose the specific learning sequence for detailed analysis, leads to the emergence of three salient questions concerning Roni's difficulties (Section 4). In the main section of the paper, we present the analysis of Roni's knowledge constructing process while he worked on the task sequence; we start with a content analysis of the task sequence (Section 5.1), discuss the role of the RBC model in the analysis (Section 5.2), and analyze Roni's work (Section 5.3). We then (Section 6) answer the three salient questions that emerged and strengthen our analysis by taking another look at PaCCs through the work of other students on the same task sequence (Section 7). We conclude the paper with a discussion (Section 8).

2. Theoretical background

2.1. Probability

2.1.1. Approaches to probability

Probability is the branch of mathematics that addresses uncertainty. Although probability theory is formally based on a set of axioms, teaching probability at school level, according to Freudenthal (1974), is based on the assumption that children have an intuitive understanding of chance concepts and are familiar with objects like coins and dice.

2.1.2. Research on learning probability

The research literature about developmental, educational and cognitive psychological aspects of learning probability is very rich (e.g., Fischbein, 1975; Fischbein and Schnarch, 1997; Jones, Langrall & Mooney, 2007; Piaget & Inhelder, 1975; Shaughnessy, 2003; Tversky and Kahneman, 1974). Because our main focus is on the process of constructing knowledge, we restrict our review to research related to the construction of knowledge in probability and to issues that address the present research context, namely binomial two-dimensional sample space, by which we mean probability situations with two discrete, binomial (i.e., yes/no) events, for example the probability of meeting a person who is larger than 1.80 m and carries a cell phone.

Quite a few research studies address the probability concepts that are prerequisite to those of the present research context (e.g., Falk, Yudilevich-Assouline & Elstein, 2012; Fischbein, Nello & Marino, 1991; Fischbein & Schnarch, 1997; Lecoutre, 1992; Nilsson, 2007; Pratt, 2000; Nunes, Bryant, Evans, Gottardis & Terlektsi, 2014). Textbooks for secondary school propose activities and models for students who do their first steps in the learning context of binomial two-dimensional sample space (e.g., Connected Mathematics Project 3, 2015; Hadas & Zaslavsky, 1996). However, we did not find research studies that analyze learning processes at that stage of learning.

Studies of knowledge construction are typically based on fine-grained analyses. In probability, such studies include the work of Pratt, Noss, and Wagner. Pratt and Noss (Pratt, 2000; Pratt and Noss, 2002) explored the microevolution of mathematical knowledge specifically focusing on how cognitive resources for probabilistic reasoning are developed and expressed in further activities. Wagner (2002, 2006, 2010) explored the process of probabilistic knowledge transfer, in which knowledge that was at first constructed for a limited context is later recognized as relevant in further problem situations.

The research reported in the present paper is based on a similarly fine-grained analysis of students' probability concepts, but differs from these studies in two respects: One, our research focuses mainly on the first emergence of students' knowledge constructs, on their components, their structure and their mutual role in the learning process. Second, we focus on the