Engineering Applications of Artificial Intelligence 67 (2018) 52–62

Contents lists available at ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

m-polar fuzzy graph representation of concept lattice

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ARTICLE INFO

Keywords:
Formal concept analysis
Fuzzy concept lattice
m-polar fuzzy set
m-polar fuzzy graph
m-polar formal fuzzy concept

ABSTRACT

Recently, the calculus of fuzzy concept lattice is studied beyond the three-way fuzzy space ($[0,1]^3$) for precise representation of uncertainty and vagueness in the attributes. However, to dovetail the uncertainty in case of voxel, multi-index or multi-polar information the properties of lattice theory need to be explored in component wise m-polar fuzzy space ($[0,1]^m$). In this case, another problem arises while finding some of the hidden or interested pattern from the given m-polar fuzzy context for the knowledge processing tasks. To conquer this problem, current paper generalizes the mathematical background of concept lattice with m-polar fuzzy sets and its graphical properties. To elicit this objective, two methods are introduced for providing a unified framework based on discovered m-polar formal fuzzy concepts and their projection.

1. Introduction

Formal Concept Analysis (FCA) is nothing but a mathematical model for data analysis and processing task (Poelmans et al., 2013; Wille, 1982). It starts the analysis from a given formal context (X, Y, R) having X as a set of formal objects, Y as a set of formal attributes, and $R \subseteq X \times Y$ as a binary relationship among them. The calculus of FCA discovers patterns in form of objects set ($A \subseteq X$) having their common attributes ($B \subseteq Y$) connected with the Galois closure operator (Ganter and Wille, 1999). The obtained object–attribute (extent–intent) pattern (A, B) is called as formal concepts or basic unit of human thought to process the knowledge. All of the discovered formal concepts can be visualized as a hierarchical order in the concept lattice via reflecting generalization and specialization among them. It means, the attributes of formal concept are inherited from the most general maximum node, while the objects are inherited from the most specific minimum node in the concept lattice. In this way, the generalized concepts contain more objects whereas the specialized concepts contain more attributes (Ganter and Wille, 1999). The properties of formal concepts and its lattice structure have been applied in several research fields for knowledge processing tasks (Bělohlávek et al., 2014; Berry and Sigayret, 2004; Bloch, 2011; Burusco and Fuentes-Gonzalez, 1994; Burusco and Fuentes-Gonzales, 2001; Prem Kumar and Aswani Kumar, 2014a; Prem Kumar and Abdullah, 2015; Prem Kumar et al., 2016; Prem Kumar, 2017). Due to this, applicability of FCA several algorithms are proposed to generate the formal concepts (Alcalde et al., 2015; Antoni et al., 2014; Aswani Kumar and Prem Kumar, 2014; Kuznetsov and Obiedkov, 2002). Furthermore, the mathematics of FCA is incorporated with various orientations like fuzzy (Bělohlávek, 2004; Bělohlávek and Konečný, 2012; Bělohlávek et al., 2014; Burusco and Fuentes-Gonzalez, 1994; Ganter and Wille, 1999; Li et al., 2015; Macko, 2013; Medina and Ojeda-Aciego, 2012), interval (Burusco and Fuentes-Gonzales, 2001; Djouadi, 2011; Djouadi and Prade, 2009; Prem Kumar and Aswani Kumar, 2012; Prem Kumar et al., 2016), bipolar (Prem Kumar and Aswani Kumar, 2014a,b), three-polar (Prem Kumar, 2017; Yao, 2016), possibility (Dubois and Prade, 2012b,a), and rough setting (Wang and Liu, 2008; Yao, 2004). To accelerate the application of FCA in dealing with uncertainty and vagueness in various research fields (Yao, 2004, 2016; Zadeh, 2008). This paper focused on the deep analysis of FCA with fuzzy setting, and its necessary conditions for the extension.

Conventionally, fuzzy decision-making is based on unipolar (positive only) input arguments. There are several approaches that allow to incorporate both positive and negative knowledge. Due to that, mathematics of FCA with fuzzy setting is extended from unipolar (i.e. interval-valued (Djouadi and Prade, 2009; Prem Kumar and Aswani Kumar, 2012; Prem Kumar et al., 2016)) to bipolar (i.e. bipolar-fuzzy setting (Prem Kumar and Aswani Kumar, 2014a,b)) and three-polar fuzzy space (Prem Kumar, 2017; Yao, 2016). These extensions are required to distinguish the positive or negative side of a given information. The positive membership degree (0, 1] of an attribute indicates that the attribute somewhat satisfies the corresponding property, and the negative membership degree [-1, 0) indicates that the attribute somewhat satisfies the implicit counter-property (Lee, 2000, 2004). The zero membership degree $\{0\}$ of an attribute means the attribute is irrelevant to the given context (Yang et al., 2013). These two sides are an integral

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https://doi.org/10.1016/j.engappai.2017.09.011
Received 26 December 2016; Received in revised form 11 August 2017; Accepted 11 September 2017
0952-1976/© 2017 Published by Elsevier Ltd.

Table 1
Some approaches on data with bipolar, three-polar and m-polar fuzzy attributes.

Reference number	Data set	Interesting pattern	Graphical visualization	Drawback of the method
Akram (2011), Akram and Dudek (2013) and Akram (2013)	Bipolar or m-polar	Given by numerical	Bipolar and hypergraph	Unable to handle multi-polar information
Alcalde et al. (2015)	Bipolar or Bicontext	Biconcepts	Concept lattice	Unable to handle multi-polar information
Alcalde et al. (2015)	Heterogeneous context	Heterogeneous formal concept	Not Given	Unable to handle multi-polar information
Chen et al. (2014)	m-polar context	Not given	Not Given	Not linked with graph and lattice
Ganter and Wille (1999)	Three-polar context	Formal concepts	Multi granulation	Takes more time for m-polar
Mesiarová-Zemanková and Ahmad (2014) and Mesiarová-Zemánková (2015)	m-polar context	Not given	Not given	No link with graph and lattice
Prem Kumar (2017)	Three-polar context	Formal concept	Concept lattice	Takes more time for m-polar
Prem Kumar (2016)	Projection of context	Formal concept	Concept lattice	Takes more time for m-polar
Yao (2016)	Three-polar context	Interval concept	Interval lattice	Takes more time for m-polar

part of the given fuzzy information which co-exist simultaneously. For example, the relationship among two organizations or a marriage couple constitute a conflict as well as a common interest side (Yang et al., 2013; Zhang and Zhang, 2004). To analyze this type of bipolar information adequate properties of bipolar fuzzy graph (Akram, 2011; Akram and Dudek, 2013; Akram, 2013; Talebi and Rashmanlou, 2014; Yang et al., 2013) has been incorporated with the concept lattice theory (Bloch, 2011; Djouadi and Prade, 2009; Dubois and Prade, 2012b,a; Prem Kumar and Aswani Kumar, 2014a,b). To handle the indeterminacy in data three-way fuzzy concept lattice is introduced (Prem Kumar, 2017) to characterize them based on truth, false and indeterminacy membership-value. In this process representing the uncertainty and vagueness in data with m-polar or multi-decision attributes addressed as a major problem. The reason is all of the approaches are seized their boundary in bipolar fuzzy space only. In this case, an alternative extension of fuzzy set and its incorporation is required for dealing with m-polar fuzzy attributes. Due to that, the current paper aimed at introducing the notation of FCA with m-polar fuzzy setting.

It can be observed that, many data set contains multi-polar or multi-decision attributes (Zadeh, 2011). The precise measurement of these multi-polar attribute is major concern for the research communities. For example, opinion of an organization with regards to another organization is based on more than two factors. Similarly, opinion of people to elect a leader in a democratic country is based on multi-criteria. The computed relationship for these types of multi-polar information is based on the point of view of their measurement beyond the bipolar or three-polar fuzzy space (Alcalde et al., 2015; Antoni et al., 2014; Aswani Kumar and Prem Kumar, 2014; Aswani Kumar et al., 2015; Li and Tsai, 2013). The computation of measurement becomes more complex when a group of friends wants to decide which movie to watch or not. Similarly, a company wants to decide which product to manufacture or not for optimum profitable. The reason is humans thinking or intelligence is beyond true, false (i.e. bipolar fuzzy space) membership value. The fact is that human thought contains m-polar independent side simultaneously to represent the given information integrally. This can be easily understand by considering the example of tug of war. In which, m people pull the rope in m-different directions. The set of people who uses the maximal strength to pull the center of given rope will move in that mth-direction. In this data set, number of people can be consider as a set of objects and, their strength can be considered as m-polar attribute. The corresponding relationship among them can be represented using the properties of m-polar (or a $[0,1]^m$) fuzzy set (Chen et al., 2014). It is a generalization of fuzzy set on X represented by a mapping $f : X \to [0,1]^m$ where $(0,0,\dots,0)$ is least element and $(1,1,\dots,1)$ is greatest element. However, for knowledge processing tasks an expert (or user) required some of the interesting pattern hidden in data with m-polar fuzzy attributes. For this purpose, a elicit connection among m-polar fuzzy set (Mesiarová-Zemanková and Ahmad, 2014; Mesiarová-Zemánková, 2015; Mesiarová-Zemánková and Hycǩo, 2015; Mesiarová-Zemánková, 2016; Obiedkov, 2012), m-polar fuzzy graph (Samanta et al., 2015), and concept lattice is required to analyze them based on super and sub concept hierarchy. This research area is still at infancy stage when compared to other extensions of FCA and their uses in knowledge processing tasks. To bridge this gap, current paper aimed at concept lattice representation using m-polar fuzzy graph as others graph theory (Bělohlávek et al., 2014; Berry and Sigayret, 2004), fuzzy graph theory (Ghosh et al., 2010), interval-valued fuzzy graph theory (Prem Kumar et al., 2016), bipolar fuzzy graph theory (Prem Kumar and Aswani Kumar, 2014a,b) and three-polar fuzzy graph (Prem Kumar, 2017) are done. This generalized representation of (fuzzy) concept lattice is required, because fuzziness is inseparable from bipolar or multi-polar truth. Less attention has been towards handling the data with m-polar fuzzy attributes though it has many applicability in various research fields. Recently, this research area received the attention of some of the researchers around the world, as given in Table 1. However, for depth analysis of these type of data set an expert need some interesting pattern for knowledge processing tasks. To fulfill this objective, an elicit connection among FCA with m-polar fuzzy setting is explored in this paper. The motivation is to visualize the data with m-polar fuzzy attributes as vertices and their corresponding relationship as nodes in the m-polar fuzzy graph for precise analysis of knowledge processing tasks. The goal is to develop a unified framework for modeling the data with m-polar fuzzy attributes precisely to accelerate the knowledge processing tasks. To achieve this goal, following problems are addressed in this paper:
(1) How to represent the data with multi-polar information in the formal context ?
(2) How to generate the m-polar fuzzy concepts ?