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Evaluating operation and coordination efficiencies of parallel-series two-stage system: A data envelopment analysis approach

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ABSTRACT

Our motivation of this study is to provide a data envelopment analysis (DEA) approach for evaluating and decomposing the operation efficiency, moreover, computing the coordination efficiency of systems with complex internal structure. We consider a two-stage system composed of three sub-systems, where the first stage is comprised of two independent sub-systems in parallel and then linked to the second stage or the third sub-system in series. Additive and multiplicative DEA efficiency measures are proposed to be jointly applied to illustrate the efficiency formation mechanism of the parallel-series system. The definition of coordination efficiency is inspired by the "gap" between accomplished optimal efficiency and idealized maximal efficiency of relevant systems. This study contributes in creating novel means to consider the different structural characteristics in the efficiency assessment of complex network systems, and to measure the externalities in terms of efficiency within their interior. The proposed models are demonstrated by revisiting the case of Taiwanese non-life insurance companies studied by Kao and Hwang. Corresponding implications of the empirical application are also discussed.

1. Introduction

Introduced by Charnes, Cooper, and Rhodes (1978), Data Envelopment Analysis (DEA) is a non-parametric mathematical approach for evaluating the relative efficiency of a set of homogenous Decision Making Units (DMUs). Without considering the internal operations or structure of DMUs, traditional DEA models typically treat DMU as a "black box" of transforming multiple initial inputs to multiple final outputs. The "black box" approaches tend to produce inaccurate efficiency scores or misleading results for systems with complex internal structure (Kao, 2014). In practice, traditional DEA models could not give the specific information regarding the sources of inefficiency within DMUs (Lewis & Sexton, 2004).

To address the limitations of traditional DEA models, Färe and Grosskopf (1996, 2000) propose a network DEA approach which considers DMU as system consisted of a network of sub-systems, some of which consume resources produced by others and some of which produce resources consumed by others (Lewis & Sexton, 2004). Among the development of network DEA, an increasing number of studies have been devoted to two-stage DEA approach in the past few years (Cook, Zhu, Bi, & Yang, 2010a). Two-stage DEA models treat efficiency evaluation problem for systems having two-stage internal structure where the initial inputs are transformed to intermediate measures (Chen & Zhu, 2004), links (Tone & Tsutsui, 2009) or intermediate flows (Mirhedayatian, Azadi, & Saen, 2014) through the first stage, and then the intermediate measures are developed into final outputs in the second stage.

Two-stage DEA simulates a general internal structure of system and provides the possibilities to assess the overall efficiency of system and decompose it into the efficiency of each sub-stage (Chen, Cook, Li, & Zhu, 2009; Kao & Hwang, 2008; Sahoo, Zhu, Tone, & Klemen, 2014), to take consideration of the cooperative and noncooperative relationships between sub-stages (Li, Chen, Liang, & Xie, 2012; Liang, Cook, & Zhu, 2008; Maghbouli, Amirteimoori, & Kordrostami, 2014), and to treat the inputs or even outputs allocation issues within the system (Chen, Du, Sherman, & Zhu, 2010; Wu, Zhu, Ji, Chu, & Liang, 2016; Yu & Shi, 2014), etc. The simplicity and representativeness of two-stage DEA approach trigger therefore significant methodological development and considerable applications in various directions (see, Cook, Liang, & Zhu, 2010; Kao, 2014a). It is obvious that simulation of system's internal structure and analysis of relations between the sub-systems are two core concerns which make two-stage DEA become one of the most active and cared approach in network DEA.

However, there are still some insufficiencies in two-stage DEA models concerning the two key points mentioned above. Firstly, two-stage DEA approach stays primarily on system's serial struc-

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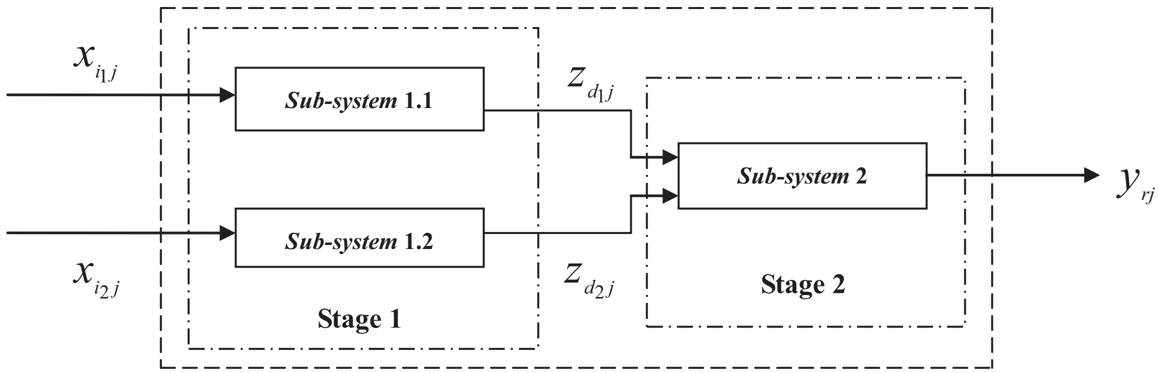

Fig. 1. Two-stage system with parallel-series internal structure.

ture, i.e. the first stage and the second stage are ordered in series and linked by intermediate measures. This feature overlooks what's going on in sub-stages: if the operations within a sub-stage are also organized in series, the system will become a multi-stages process, and its efficiency evaluation lies outside the range of two-stage DEA, for example the research of Kao (2014b); but if the internal operations of a sub-stage are in parallel, the general internal structure of system is still two-stage, then how to evaluate and decompose the efficiency of system with respecting the different efficiency formation mechanisms? Secondly, the relations between sub-stages are mostly studied in cooperative or non-cooperative perspective by two-stage DEA (Liang et al., 2008; Liang, Yang, Cook, & Zhu, 2006). Nevertheless, this perception does not concern about the coordination within the system which is a fundamental relationship between sub-stages or even within the sub-stages. The coordination reflects an important characteristic indeed, i.e. the "tradeoff" inside the systems with complex internal structure. How to evaluate the performance of coordination in the two-stage system and what are the implications of this measurement?

This paper attempts to deal with the issues mentioned above by developing parallel-series DEA models to measure and decompose efficiency of the system that has a two-stage network comprised of three sub-systems, where the two formers form a parallel structure in the first stage and then connected to the latter or the second stage in series. We propose also to quantify the coordination performance of the system by measure of coordination efficiency, which we define as the ratio of the accomplished optimal efficiency score over the idealized maximal efficiency score of the relevant systems or stages. The conventional two-stage DEA models are not suitable for evaluating the efficiency of our proposed system, because the parallel-series structure is different from the two-stage process previously studied. Therefore, we propose to combine the additive efficiency decomposition method of Chen et al. (2009) and the multiplicative method of Kao and Hwang (2008) to clarify the difference between the efficiency formation mechanism of parallel system and that of serial system, respectively. Then, based on the efficiency scores evaluated by the proposed models, we estimate the coordination efficiencies for each sub-system, each stage and the system as whole, and evaluate the "externality" produced by coordination within the system.

The remainder of this paper is organized as follows. Section 2 presents the mathematical details of the parallel-series two-stage DEA approach with its efficiency decomposition procedure. Section 3 introduces the method of coordination efficiency measurement for parallel-series two-stage system. In Section 4, our proposed approach is applied to the data set used in Kao and Hwang (2008) to verify the applicability of the models. Section 5 outlines conclusions and future research directions.

2. Model description and efficiency decomposition

2.1. Two-stage parallel-series system and CCR efficiency

Consider a two-stage system composed of three sub-systems as shown in Fig. 1, for each of n homogeneous DMUs denoted by $\text{DMU}_j (j = 1, 2, ..., n)$. The first stage is comprised of two independent sub-systems in parallel where sub-system 1.1 uses I_1 inputs $x_{i_1 j} (i_1 = 1, 2, ..., I_1)$ to produce D_1 outputs $z_{d_1 j} (d_1 = 1, 2, ..., D_1)$, and sub-system 1.2 consumes I_2 inputs $x_{i_2 j} (i_2 = 1, 2, ..., I_2)$ to generate D_2 outputs $z_{d_2 j} (d_2 = 1, 2, ..., D_2)$. The first stage is then linked to the second stage or sub-system 2 in series by the outputs from the first stage $z_{d_1 j} (d_1 = 1, 2, ..., D_1)$ and $z_{d_2 j} (d_2 = 1, 2, ..., D_2)$, referred to as intermediate measures (Chen & Zhu, 2004) or links (Tone & Tsutsui, 2009). The second stage employs both the intermediate measures to yield S final outputs $y_{rj} (r = 1, 2, ..., S)$.

Let v_{i_1} and v_{i_2} denote the weights on the inputs to sub-system 1.1 $x_{i_1 j} (i_1 = 1, 2, ..., I_1)$ and sub-system 1.2$x_{i_2 j} (i_2 = 1, 2, ..., I_2)$, respectively. As the intermediate measures play dual role in the first stage and in the second stage, we denote $u^1_{d_1}$ and $u^1_{d_2}$ as the weights on the outputs flowing from the first stage, and $u^2_{d_1}$ and $u^2_{d_2}$ as the weights on the intermediate measures entering the second stage. The weight u_r is given to the final outputs $y_{rj} (r = 1, 2, ..., S)$.

According to the classic CCR model proposed by Charnes et al. (1978), the efficiency of DMU_0 is defined as the maximum of a ratio of the weighted sum of final outputs to the weighted sum of initial inputs, and subject to the condition that the same ratio for all DMUs must be less than or equal to one. The CCR efficiency of the parallel-series system depicted in Fig. 1 for DMU_0, denoted as θ_0^{CCR}, can be calculated by the following model (1):

$$\theta_0^{CCR} = \max \frac{\sum_{r=1}^{S} u_r y_{r0}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}}$$
$$\text{s.t.} \quad \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \le 1, \quad j = 1, 2, ..., n$$
$$u_r, v_{i_1}, v_{i_2} \ge 0, \ r = 1, 2, ..., S, \ i_1 = 1, 2, ..., I_1, \ i_2 = 1, 2, ..., I_2. \quad (1)$$

where v_{i_1}, v_{i_2}, u_r are the weights on inputs x_{i_1}, x_{i_2} and outputs y_r, respectively. The model (1) can be transformed into the following linear programming program (2) by using the Charnes–Cooper transformation (Charnes & Cooper, 1962):