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Dynamic output-feedback control for positive Roesser system under the switched and T-S fuzzy rules

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ABSTRACT

This paper is concerned with the dynamic output-feedback controller design for the positive Roesser type nonlinear system, which is intrinsically characterized by the switched mechanism and subtly decomposed into the linear form under the Takagi–Sugeno fuzzy rules. Firstly, based on the co-positive Lyapunov function and the average dwell time method, sufficient conditions are presented, under which the resulting closed-loop system is exponentially stable and has l_1-gain bound γ. Then, explicit expressions are also given to derive the expected controller gain matrices with desired l_1-gain bound. Finally, effectiveness of the proposed results is illustrated by two numerical examples.

1. Introduction

In the past few decades, two-dimensional (2-D) systems have drawn much research attention since a wide variety of practical systems can be described by such kind of 2-D models, such as those in image data processing and transformation, water stream heating, thermal processes, gas absorption and so on [10]. On another front, many systems involving in epidemiology, ecology and biology have the special characteristic that their states and outputs are nonnegative whenever both the initial state and the system inputs are nonnegative. This special class of systems are called positive systems [2,5,16,17], which have been discussed extensively in the literature, see [11,21,24,43] and the references cited therein for example. Especially, dynamic output-feedback control problem has been considered in [6] for the continuous-time positive systems, and the stability issue of positive switched systems has been tackled in [31,39]. However, due to the fact that the state variables are all constrained on a cone and many well-established methodologies dealing with the general systems are not easily adapted for the positive ones [29], usually, it is difficult to analyze the positive models, and most of the results existed for the positive systems are mainly focused on the case of one-dimensional (1-D) ones, whereas the corresponding parts on the 2-D positive systems are relatively few [8]. Hence, the study of 2-D positive systems is a challenging while necessary problem.

It is well known that practical systems are always subjected to abrupt changes either in their structures or in the parameters [37]. To characterize such changes, one effective method is to introduce the switched mechanisms [23,30,38]. As a special class of hybrid systems, switched models consist of a family of subsystems and a logical rule that governs the switching among them. Stability analysis and synthesis for switched systems are one of the hot research topics in the past decades [9,22,28,32,44], where the popular investigation methods include the multiple Lyapunov functions approach [12,45] and the

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average dwell time (ADT) technique [27,42]. Especially, the feedback stabilization problem for switched systems is more challenging since both the control input and the switching law are simultaneous design variables, and their interactions must be fully understood [33]. When stabilizing the switched systems, in general, there are three types of feedback control schemes: state-feedback control, static output-feedback control and dynamic output-feedback control. It is worth noting that the state variables of many practical plants cannot be measured directly, which means that the state-feedback controller can be designed only if a state observer can be designed firstly, and this will not only increase the cost but also decrease the system reliability. Therefore, the output-feedback controller is more preferable when designing controller to ensure the desired performance for the closed-loop system.

The concept of fuzzy set has been first introduced by Zadeh [40] which accords with the thinking characteristic of human brain. In general, the fuzzy set can be classified into two types, where the type-2 fuzzy set is usually an extension of the ordinary fuzzy set which is always called type-1 fuzzy set. The membership grade for each element of the type-2 fuzzy set is also a fuzzy set in [0, 1], whereas it is a precise number for the type-1 fuzzy set [25]. Novel results concerning the type-2 fuzzy models can be found in [1,15,26]. On the other hand, the famous Takagi–Sugeno (T-S) fuzzy models, which has been put forward by Takagi and Sugeno in 1985 [34], can approximate the nonlinear system by utilizing a set of linear subsystems combined with nonlinear fuzzy membership functions. To be more specific, in the convex compact region, the T-S fuzzy model has been proved that it can approximate the smooth nonlinear function with any degree of accuracy [13], which makes it possible to use the methodology for linear systems to deal with the nonlinear ones, especially for those systems that are highly complex and nonlinear [20]. Due to such kind of merits, in the last three decades, T-S fuzzy synthesis have attracted the attention of many researchers [3,7,14,19,35,36,41]. Motivated by the aforementioned discussions, in this paper, the dynamic output-feedback controller will be designed for the 2-D switched positive nonlinear systems such that the resulting closed-loop system is exponentially stable and has a prescribed l_1-gain bound γ.

In this paper, considering the characteristic of positive systems, a novel co-positive Lyapunov function is constructed and then the exponential stability is guaranteed by restricting the switching signal satisfying the ADT constraint, i.e., the *nonlinear 2-D positive* system can achieve stability on the condition that the subsystems do not switch very frequently in the sense of average time. *Compared with the previous works, the main contributions of the present article are three-fold. 1) This is the first few attempt to analyze the exponential stability for positive Roesser systems by considering both the switched mechanism and the T-S fuzzy rules simultaneously. 2) An l_1-norm induced performance index is presented to characterize the effect of extraneous disturbance on the system output on a whole. 3) Dynamic output-feedback controller is also designed which assures the closed-loop 2-D system not only to be positive but also having an l_1-gain bound γ.*

The remainder of this paper is organized as follows. In Section 2, the problem under consideration is formulated, and a 2-D dynamic output-feedback fuzzy controller is proposed. In Section 3, sufficient conditions are presented to guarantee the resulting closed-loop system being exponentially stable. The first part of Section 4 is about the l_1-gain analysis and the second part focuses on the dynamic output-feedback controller design. Two numerical examples are given in Section 5 to demonstrate the effectiveness of the obtained results. Conclusions are drawn in Section 6.

Notation. The notations used in this paper are fairly standard. $\mathbb{R}^{n\times m}$ stands for the set of all real $n\times m$ matrices, and $\mathbb{R}_+^{n\times m}$ is the set of all $n\times m$ matrices with nonnegative entries. $\mathbb{R}^n=\mathbb{R}^{n\times 1}$ denotes the set of n-dimensional column vectors, and the set of n-dimensional column vectors with nonnegative entries will be denoted by $\mathbb{R}_+^n$, if further all the entries are positive, it is called strictly positive and denoted as $\mathbb{R}_{++}^n$. The superscript "T'" means the matrix transposition. diag$\{\cdots\}$ denotes a diagonal matrix and the set of nonnegative integers will be denoted by $\mathbb{Z}_+$. $A\prec 0$ means that matrix A is negative definite, and $A\geq 0$ (<0) means that all entries of matrix A are nonnegative (negative). $[A]_{i,j}$ stands for the element located at the ith row and the jth column of matrix A. The Moore–Penrose inverse of a real matrix A is denoted as A_g, namely, A_g is the Moore–Penrose inverse of A if it satisfies $AA_gA=A$, $A_gAA_g=A_g$, $(AA_g)^T=AA_g$ and $(A_gA)^T=A_gA$. I_n is the identity matrix with $n\times n$ dimension. The 1-norm of a 2-D vector $x(k,l)=(x_1(k,l),x_2(k,l),\ldots,x_n(k,l))^T\in\mathbb{R}^n$ with $k,l\in\mathbb{Z}_+$ is defined as $\|x(k,l)\|_1\triangleq\sum_{s=1}^n|x_s(k,l)|$. For a vector-valued function $x:\mathbb{Z}_+\times\mathbb{Z}_+\to\mathbb{R}^n$, its l_1-norm is defined as $\|x\|_{l_1}\triangleq\sum_{k=0}^\infty\sum_{l=0}^\infty\|x(k,l)\|_1$, and we say $x\in l_1\{[0,\infty)\times[0,\infty)\}$ if $\|x\|_{l_1}<\infty$.

2. Problem formulation and preliminaries

Consider the following Roesser type switched nonlinear system

$$\begin{cases} x^+(k,l)=f_1^{\sigma(k,l)}(x(k,l),w(k,l),u(k,l)),\\ z(k,l)=f_2^{\sigma(k,l)}(x(k,l),w(k,l),u(k,l)),\\ y(k,l)=f_3^{\sigma(k,l)}(x(k,l),w(k,l)), \end{cases} \tag{1}$$

where

$$x^+(k,l)=\begin{pmatrix}x^h(k+1,l)\\ x^v(k,l+1)\end{pmatrix},\quad x(k,l)=\begin{pmatrix}x^h(k,l)\\ x^v(k,l)\end{pmatrix},$$

in which $x^h(k,l)\in\mathbb{R}^{n_1}$, $x^v(k,l)\in\mathbb{R}^{n_2}$ and $x(k,l)\in\mathbb{R}^n$ are the horizonal state vector, the vertical state vector and the whole state vector at point $(k,l)\in\mathbb{Z}_+\times\mathbb{Z}_+$, respectively. $w(k,l)\in\mathbb{R}^{m_1}$ is the disturbance input, $u(k,l)\in\mathbb{R}^{m_2}$ is the control input, $z(k,l)\in\mathbb{R}^{r_1}$ is the controlled output, and $y(k,l)\in\mathbb{R}^{r_2}$ is the measured output. $\sigma(\cdot,\cdot):\mathbb{Z}_+\times\mathbb{Z}_+\to\underline{\mathbf{N}}$ is a switching signal,