Information Sciences 422 (2018) 161–176

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A decision-making model based on interval additive reciprocal matrices with additive approximation-consistency

Fang Liu[a,*], Ya-Nan Peng[a], Qin Yu[a], Hui Zhao[b]

[a] School of Mathematics and Information Science, Guangxi University, Nanning, Guangxi 530004, China
[b] China-ASEAN Research Institute, Guangxi University, Nanning, Guangxi 530004, China

ARTICLE INFO

Article history:
Received 11 January 2017
Revised 12 August 2017
Accepted 5 September 2017
Available online 6 September 2017

Keywords:
Decision making
Interval additive reciprocal matrix
Additive approximation-consistency
Exchange method
Interval weight

ABSTRACT

In order to capture the rationality experienced by decision makers in choosing the best alternative(s), of much interest is to study the consistency of preference relations in the analytic hierarchy process (AHP). When the typical AHP is extended by using fuzzy numbers to evaluate the opinions of decision makers, the consistency of fuzzy judgments is worth to be considered. In this paper, we analyze some definitions of interval additive reciprocal matrices with additive consistency. It is concluded that interval additive reciprocal matrices are inconsistent in essence. The concept of additive approximation-consistency of interval additive reciprocal matrices is proposed. Moreover, a novel exchange method is designed to enumerate all permutations of alternatives for checking the approximation-consistency. By considering the randomness exhibited in pairwisely comparing alternatives, a method of obtaining the interval weight vector is given. A new algorithm of solving the decision making problem with interval additive reciprocal matrices is proposed. Finally, two numerical examples are carried out to illustrate the new definition and some comparisons are offered.

1. Introduction

When one is faced with a complex decision making pursuit, a feasible method is helpful to achieve a rational and reasonable decision. The hierarchical approach proposed by Saaty [25,26] has been widely used to reduce a complex system or planning process to a hierarchy of criteria, sub-criteria and alternatives. By pairwisely comparing alternatives, the opinions of decision makers are expressed as preference relations. Then they are used to derive the weights of alternatives and the best alternative(s) is(are) chosen. In the typical AHP, multiplicative reciprocal preference relations are obtained by evaluating the comparison ratios in relative measurements by virtue of the scale from 1 to 9. Moreover, based on fuzzy set theory [44], the preference degree of alternative x_i over alternative x_j can be expressed as $r_{ij} \in [0, 1]$. Then a fuzzy binary relation on $X = \{x_1, x_2, \dots, x_n\}$ is defined and a preference matrix with the entries r_{ij} is given. The preference matrix is called as a fuzzy preference relation [24,28], which is further recalled as an additive reciprocal preference relation [19] due to the property $r_{ij} + r_{ji} = 1$. It is seen from the definition of additive reciprocal matrices that the preference intensities r_{ij} are exact real numbers. However, owing to the complexity and uncertainty of the real-world decision making problems, it is difficult to

* Corresponding author .
E-mail addresses: fang272@126.com, f_liu@gxu.edu.cn (F. Liu), 815926672@qq.com (Y.-N. Peng), 282007731@qq.com (Q. Yu), aseanzhao@126.com (H. Zhao).

http://dx.doi.org/10.1016/j.ins.2017.09.014

provide precise preference values to evaluate the judgments. Along this line, Saaty and Vargas [27] proposed that interval numbers can be used to capture the uncertainty experienced by decision makers when making pairwise comparisons. Then an interval multiplicative reciprocal preference relation was defined where the scale from 1/9 to 9 was used. Following the idea of Saaty and Vargas [27], it is popular to study decision making models and their applications, where the judgements of decision makers are expressed as interval-valued comparison matrices [9,12,37,48]. In particular, it is noted that various decision making models based on interval fuzzy preference relations have been proposed [9,37,47]. Referring to the terminology of interval multiplicative reciprocal preference relations, hereafter interval fuzzy preference relations are recalled as interval additive reciprocal preference relations [19].

In order to avoid the self-contradiction of the opinions, the consistency and transitivity of preference relations are important according to the typical AHP [26]. For a multiplicative preference relation $A = (a_{ij})_{n\times n}$, its consistency means that the relations $a_{ij} = a_{ik} \cdot a_{kj}$ $(\forall i, j, k = 1, 2, \ldots, n)$ are satisfied [26]. An additive reciprocal matrix $R = (r_{ij})_{n\times n}$ with additive consistency and multiplicative consistency means the transitivity of $r_{ij} = r_{ik} - r_{jk} + 0.5$ and $r_{ij} \cdot r_{jk} \cdot r_{ki} = r_{ji} \cdot r_{ik} \cdot r_{kj}$ $(\forall i, j, k = 1, 2, \ldots, n)$, respectively [28]. In addition, the cardinal consistency of additive reciprocal matrices was modeled by using a functional equation in [4]. Unfortunately, when the real-valued judgements are extended to interval-valued ones, the consistency of the comparison matrices cannot be defined by directly extending the definitions of consistent multiplicative reciprocal matrices [7]. Recently, Liu et al. [16] have analyzed the existing definitions of consistent interval multiplicative reciprocal matrices. It has shown that interval multiplicative reciprocal matrices are inconsistent in essence. The new concept of approximation-consistency of interval multiplicative reciprocal preference relations has been proposed. Moreover, it is noted that some consistency indexes have been proposed to measure the inconsistency degree of interval-valued reciprocal matrices [5,6]. As compared to consistency definitions, the consistency indexes mainly give a value to a comparison matrix so that the deviation degree from a consistent matrix can be quantified [26]. The consistency definition of a comparison matrix is to give sufficient conditions to address the ideal case with strict logic and rationality. Following the idea in [16], interval additive reciprocal matrices are the softened versions of additive reciprocal preference relations. The conditions of additive reciprocal matrices with additive consistency and multiplicative consistency cannot be extended directly to define the consistency of interval additive reciprocal matrices. It is concluded that interval additive reciprocal matrices are inconsistent in nature. In the literature, one can see that there are various consistency definitions of interval additive reciprocal matrices [18,35,39,40]. The consistency indexes have also been proposed to address the inconsistency degree of interval additive reciprocal matrices [6]. The above observations motivate us to clarify the existing consistency definitions of interval additive reciprocal matrices along with comparing the consistency indexes. In this study, some definitions of interval additive reciprocal matrices with additive consistency are reviewed comprehensively. It is shown that a weak-consistency definition of interval additive reciprocal matrices should be given to capture the limit rationality of decision makers. A new concept of additive approximation-consistency of interval additive reciprocal matrices is proposed. In order to check the additive approximation-consistency of interval additive reciprocal matrices, a novel exchange method of enumerating the permutations of alternatives is proposed. Furthermore, it is important how to derive the weight vector from interval additive reciprocal matrices. One can see that many methods have been proposed in the literature, such as the C-OWA operator method [38], the convex combination method [18,43], the eigenvector method [21], the goal programming models [8,30,31,36,45,46] and others. Here we consider the randomness exhibiting in pairwisely comparing alternatives and give a new method to obtain the interval weights. Finally, a new algorithm for solving decision making problems with interval additive reciprocal matrices is proposed.

The structure of this paper is shown as follows. Section 2 analyzes the definitions of interval additive reciprocal preference relations with additive consistency and compares the consistency indexes. In Section 3, a new concept of additive approximation-consistency of interval additive reciprocal matrices is proposed by considering the permutations of alternatives. The corresponding properties are further studied in detail. In Section 4, we give a novel exchange method to enumerate the $n!/2$ permutations of n alternatives. The method of obtaining the interval weights is given under the consideration of the randomness exhibited in pairwisely comparing alternatives. Section 5 offers a new algorithm to the decision-making problem with interval additive reciprocal comparison matrices. Two illustrative examples are carried out to illustrate the new definition and algorithm. The main conclusions and research prospects are covered in Section 6.

2. Reviews on some definitions of interval additive reciprocal matrices with additive consistency

In the section, let us review the known definitions of interval additive reciprocal matrices with additive consistency and compare the consistency indexes. First, it is interest to recall the idea of defining the consistency of comparison matrices. In relative measurements, when one gives $A = 3B$ and $B = 2C$, the consistency of the judgements implies $A = (2 \times 3) \cdot C = 6C$ [26]. Conversely, if a decision maker does not give $A = 6C$, the judgments are considered to be inconsistent. Moreover, when the preference intensities of alternatives are expressed as real numbers in [0, 1], one has the binary relation $(A, B) \to r \in [0, 1]$. For instance, if we have $(A, B) \to 0.8$ and $(B, C) \to 0.3$, the consistency of the judgments indicates $(A, C) \to (1.5 - 0.8 - 0.3) = 0.4$ [11]. For the set of alternatives $X = \{x_1, x_2, \ldots, x_n\}$, a series of pairwise comparisons are made and an additive reciprocal preference relation $B = (b_{ij})_{n\times n}$ is defined as follows:

Definition 1. [28] $B = (b_{ij})_{n\times n}$ is called as an additive reciprocal preference relation, if b_{ij} is the preference intensity of alternatives x_i over x_j with $b_{ij} + b_{ji} = 1$, $0 \le b_{ij} \le 1$, and $b_{ii} = 0.5$ for $i, j = 1, 2, \ldots n$.