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Characterizing reducts in multi-adjoint concept lattices☆

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ABSTRACT

The construction of reducts, that is, minimal sets of attributes containing the main information of a database, is a fundamental task in different frameworks, such as in Formal Concept Analysis (FCA) and Rough Set Theory (RST). This paper will be focused on a general fuzzy extension of FCA, called multi-adjoint concept lattice, and we present a study about the attributes generating meet-irreducible elements and on the reducts in this framework. From this study, we introduce interesting results on the cardinality of reducts and the consequences in the classical case.

1. Introduction

Nowadays, collected databases generally contain a large amount of data which makes their treatment a really difficult task. In addition, these data usually include redundant information that only serves to increase the complexity to handle the information. The reduction of data, preserving the main information in the considered database, is a key step in many areas that consider databases, for example, software engineering [52], information retrieval [22,33], data mining [46,47], knowledge discovery [25,35,42], machine learning [37,55], fuzzy rough set theory [20,21,30,39], among others.

Formal concept analysis [27] is considered a useful tool to treat information contained in databases by using a mathematical structure called concept lattice. These databases are composed by sets of attributes A and objects B related between them by means of a relation $R \subseteq A \times B$. The high complexity in the computation of the concept lattice makes natural to consider mechanisms in order to reduce the set of attributes or objects in the considered database. This important goal in FCA has been studied in diverse papers, which introduce different reduction mechanisms trying to preserve the main information [5,12,26,34,36,39,44,46,47,54,56].

In [15], the attribute classification theorems based on the categorization of the set of attributes given by Pawlak in RST [43] were presented. These theorems were stated within the fuzzy framework of multi-adjoint concept lattices [40,41], which is a generalization of FCA that provides a more flexible environment capable of accommodating other fuzzy approaches given in the literature [1,2,7,10,28,31]. Specifically, the attribute classification theorems divide into three types the set of attributes—absolutely necessary, relatively necessary and absolutely unnecessary attributes—remaining the process of selection of these attributes for the construction of reducts.

Another important task in FCA is the reduction of the size of the concept lattice which is closely related to the context reduction. Although the philosophy of the attribute and size reduction methods are different, they are not completely independent since both mechanisms can be combined. Indeed, we can find several papers where the size of the concept lattice

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is reduced by means of a previous reduction of the context. For example, a combination of attribute and size reduction mechanisms in the multi-adjoint concept lattice framework was given in [19]. Therefore, the computation of reducts will also be useful for the different size reduction mechanisms.

Some of the most interesting papers dealing with this kind of reduction method will be mentioned in the following. In [33], a size reduction of the concept lattice is carried out by using fuzzy k-means clustering to reduce the attribute-object matrix. Moreover, the influence of this kind of reduction on the association rules mining is studied in [32]. In [23], three reduction techniques are analyzed, in particular, simplification techniques involve the reduction both the number of the concepts appearing in the lattice and the number of objects and attributes in the context; selection techniques always simplify the size of the lattice and occasionally entail the reduction of the context. Furthermore, the size of the concept lattice is reduced in [51] considering different levels of information granules over the attributes of the context. Finally, the use of thresholds and hedges also provides interesting methodologies for reducing the size of the concept lattices [8,9,18].

In this paper, our research topic will be focused on the attribute reduction preserving the original concept lattice structure, specifically on the study of reducts of any multi-adjoint context. For that purpose, we introduce a new definition related to the attributes that generate meet-irreducible concepts of a multi-adjoint concept lattice. From this notion, we rewrite in a simpler way the attribute classification theorems, making easier their application, and we present several properties relating this notion to the relative necessary attributes. In addition, a study about the cardinality of reducts will be introduced. We will prove that when the set of relatively necessary attributes is non-empty, reducts with different cardinalities arise. In this study, we will show under what conditions the reducts of a multi-adjoint context have the same cardinality. Moreover, we will provide a bound of the cardinality of any reduct, together with more interesting results. The introduced results will be considered in the classical case providing interesting consequences.

Although the paper is focused on the multi-adjoint concept lattice framework, all these results can be interesting in order to compute the reducts in other (fuzzy) FCA frameworks. Specifically, as we commented previously, the study given in this paper could be very useful in all the size reduction mechanisms of formal contexts previously cited [23,32,33,51]. In addition, this paper can be also applied to other approaches of FCA. For instance, the truth value set considered in interval-valued [3,11,49], bipolar [50] and complex vague [48] concept lattices forms a complete lattice and therefore, the attribute reduction and their corresponding properties presented here can be taken into consideration in all these settings.

On the other hand, it is also important to mention that there exist different real-life applications where reducts have proved to be very useful sets. For example, reducts are used in [6] in order to obtain spacial fuzzy decision rules to analyze social and environmental causes of neural tube birth defects. They can be used as a mechanism in order to reduce the systems of fuzzy relation equations and simplify the computation of their solutions [24]. The helpfulness of reducts is also shown in [57] where these sets are applied to English letter handwriting recognition. Moreover, a study on relationships between reducts in classical FCA and RST has been given in [45], where the properties of reducts in the classical case presented in Section 4 can be applied. All the above mentioned underlines the close relation between these two theories. A complete study on the impact of the results introduced in this work on the fuzzy RST framework will be analyzed in the future.

The paper is organized as follows: an overview associated with preliminary notions of multi-adjoint concept lattice framework and attribute reduction are recalled in Section 2. Section 3 presents several properties about attributes that generate meet-irreducible elements of a concept lattice. This section also includes some results related to the cardinality of reducts together with different examples. The consideration of the previous results in the classical case is given in Section 4. Section 5 finishes with several conclusions and future challenges.

2. Preliminaries

In this section we recall the basic notions and necessary results in order to classify the attributes in the multi-adjoint concept lattice framework. Examples related to these preliminary notions can be found in [13–15,40,41].

2.1. Multi-adjoint concept lattice framework

Adjoint triples are the basic computational operators [13,16,17] in the considered fuzzy concept lattice framework. These operators are generalizations of a triangular norm (t-norm) and its residuated implication [29].

Definition 1. Let $(P_1, \leq_1)$, $(P_2, \leq_2)$, $(Q, \leq_Q)$ be posets and $\& : P_1 \times P_2 \to Q$, $\swarrow : Q \times P_2 \to P_1$, $\nwarrow : Q \times P_1 \to P_2$ be mappings, then $(\&, \swarrow, \nwarrow)$ is an *adjoint triple* with respect to P_1, P_2, Q if:

$$x \leq_1 z \swarrow y \quad \text{iff} \quad x \& y \leq_Q z \quad \text{iff} \quad y \leq_2 z \nwarrow x \tag{1}$$

where $x \in P_1$, $y \in P_2$ and $z \in Q$. The condition (1) is also called *adjoint property*.

Once we have recalled this notion, the definitions of multi-adjoint frame and context are given below.

Definition 2. A *multi-adjoint frame* is a tuple $(L_1, L_2, P, \&_1, \dots, \&_n)$ where $(L_1, \preceq_1)$ and $(L_2, \preceq_2)$ are complete lattices, $(P, \leq)$ is a poset and $(\&_i, \swarrow^i, \nwarrow_i)$ is an adjoint triple with respect to L_1, L_2, P, for all $i \in \{1, \dots, n\}$.