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Further studies on H_∞ observer design for continuous-time Takagi–Sugeno fuzzy model

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ABSTRACT

This brief paper is concerned with the problem of observer-based H_∞ control for nonlinear systems presented by T–S (Takagi–Sugeno) fuzzy models. The aim is to reduce the conservatism of finding a smaller H_∞ performance index by applying the non-quadratic Lyapunov function. The one-step LMIs (Linear Matrix Inequalities) method is applied to design the controller gains and observer gains. In addition, the initial positions of the observer states are also discussed and an optimization problem is solved to obtain new stable region. In the end, three examples are given to demonstrate the effectiveness of the proposed approach.

1. Introduction

In recent years, considerable attention has been paid to the analysis and synthesis of nonlinear systems [3,6,7,16,17,38,42,43], among them, the fuzzy control is an efficient method. Fuzzy control is mainly based on Lyapunov theory and the construction of the Lyapunov function is very important, till now, various kinds of Lyapunov function have been developed such as [8,14,16,17,25–29,39,40] and the references therein. Using the quadratic Lyapunov function means a positive definite matrix P must be found to satisfy the Lyapunov equations or the LMIs for all the local models. However, the positive definite matrix P might not exist for all the local models, especially for highly nonlinear complex systems. Then, in order to reduce the conservatism arisen from the using of single Lyapunov matrix, the non-quadratic Lyapunov function is constructed [13]. But, it should be noted that if the Lyapunov function depends on the membership function, the time derivatives of the membership functions have to be dealt with for continuous-time fuzzy systems. Since the membership functions are composed with premise variables, the analysis of the premise variable is the key step. There are mainly two cases of the premise variables: (1) The premise variables are independent of the system states; (2) The premise variables depend on the system states. For the first case, the bound of the time derivatives of the membership functions can be set beforehand, however, for the second case, the bounds of the derivatives can not be set beforehand because it often contains the input which is unknown before the design of the controller [15,24,36,37].

Fuzzy observer design was first studied in [30], then the separation theorem of fuzzy observer and controller was proposed in [23] but it can not be used for uncertain fuzzy systems. It was first discovered in [19] that the introduced outer

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variables X_{ij} $(i \neq j)$ need not be symmetrical but the observer was designed by applying two-step LMIs method. The conservatism of the two-step LMIs method was reduced in [20] by proposing a one-step LMIs method which has the similar property with separation theorem and has been extended to deal with time-delays [5,21,22]. There are also some results about unknown premise variables such as [1,2,33,41]. Note that the results in [19–22] were based on the quadratic Lyapunov function and it was shown in [21] that the conservatism of the results in [20–22] can be further reduced by applying the non-quadratic Lyapunov function such as [13,24,31]. Recently, the non-quadratic Lyapunov function is used in [34] and [35] to design the observer for unknown and known premise variables respectively. However, an inequality has to be used many times which leads to too many parameters. Searching these parameters is a time-consuming work and a source of conservatism.

Motivated by the foregoing discussions, the problem of observer-based control is further studied in this brief paper. The main contributions of this paper can be summarized as follows: (1) The problem that how to use the non-quadratic Lyapunov function to design the observer in one step is solved here; (2) The initial positions of the observer states are also discussed and new stable region which contains the initial positions is obtained by solving an optimization problem. Detailedly, the non-quadratic Lyapunov function is constructed and two lemmas are obtained to show that the controller gains and observer gains can be designed respectively or simultaneously. Then, in order to get some LMIs, the time derivatives of the membership function are analyzed. A region satisfying the H_∞ performance requirement is obtained by searching the bounds of the derivatives. In addition, the initial positions of the observer states are discussed and an optimization problem is solved to ensure the overall system is stable. This brief paper can be seen as a complement to [21] and the simulation shows that when the method in [21] fails for some examples, this paper can still get some satisfying results.

In this paper, the notation A^T denotes the transpose of A. A star $*$ in a symmetric matrix denotes the transposed element in the symmetric position.The symbol I stands for the identity matrix with proper dimensions. For $F \in \mathbb{R}^{m\times n}$, F_i denotes the ith matrix of a series of matrices.

2. Preliminaries and backgrounds

Let us consider the following well known T–S fuzzy model in [19–22] which can be obtained by using sector nonlinear method in [32],

$$\begin{aligned} \dot{x} &= A_h x + B_h u + N_h \omega, \\ z &= C_h x + D_h u, \\ y &= E_h x, \end{aligned} \tag{1}$$

where $A_h = \sum_{i=1}^{r} h_i(\theta)A_i$, $B_h = \sum_{i=1}^{r} h_i(\theta)B_i$, $N_h = \sum_{i=1}^{r} h_i(\theta)N_i$, $C_h = \sum_{i=1}^{r} h_i(\theta)C_i$, $D_h = \sum_{i=1}^{r} h_i(\theta)D_i$, $E_h = \sum_{i=1}^{r} h_i(\theta)E_i$ and $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $N_i \in \mathbb{R}^{n\times s}$, $C_i \in \mathbb{R}^{\nu\times n}$, $D_i \in \mathbb{R}^{\nu\times m}$, $E_i \in \mathbb{R}^{w\times n}$ are known matrices, $\omega \in \mathbb{R}^{s}$ is the exogenous disturbance input, $z \in \mathbb{R}^{\nu}$ is the controlled output, $y \in \mathbb{R}^{w}$ is the measured output,

$$h_i(\theta) = h_{1+i_1+i_2\times 2+\cdots+i_p\times 2^{p-1}} = \prod_{j=1}^{p} w^j_{i_j}(\theta_j), i \in \{1,\ldots,2^p\}, i_j \in \{0,1\},$$

$$w^j_0(\cdot) = \frac{\overline{nl_j} - nl_j(\cdot)}{\overline{nl_j} - \underline{nl_j}}, w^j_1(\cdot) = 1 - w^j_0(\cdot), j \in \{1,\ldots,p\},$$

θ is the known premise variable bounded and smooth in the compact set $\mathbb{C}$ where the nonlinear system can be described as T–S model (1), $\mathbb{C} = \bigcap_q \{x : |g_q x| \le \varepsilon_q\}, q = 1,\ldots,n$, ($g_q \in \mathbb{R}^{1\times n}$, only the element in $(1, q)$ is one and the others are zeros, for example, if $n = 3$, we have $g_1 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$, $g_2 = \begin{bmatrix}0 & 1 & 0\end{bmatrix}$, $g_3 = \begin{bmatrix}0 & 0 & 1\end{bmatrix}$).

The observer-based controller is designed as

$$\dot{\hat{x}} = A_h\hat{x} + B_h u + T(h)\big(y - \hat{y}\big), \hat{y} = E_h\hat{x}, u = K(h)\hat{x}, \tag{2}$$

where $K(h)$ and $T(h)$ are the controller gains and observer gains to be designed. Let us combine (1) with (2), the closed-loop fuzzy system is written as

$$\dot{\tilde{x}} = G(h)\tilde{x} + M(h)\omega, z = H(h)\tilde{x}, \tag{3}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x - \hat{x} \end{bmatrix}, G(h) = \begin{bmatrix} A_h + B_h K(h) & -B_h K(h) \\ 0 & A_h - T(h)E_h \end{bmatrix},$$

$$M(h) = \begin{bmatrix} N_h \\ N_h \end{bmatrix}, H(h) = \begin{bmatrix} C_h + D_h K(h) & -D_h K(h) \end{bmatrix}.$$