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Geometrical convergence rate for distributed optimization with time-varying directed graphs and uncoordinated step-sizes

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ABSTRACT

This paper studies a class of distributed optimization algorithms by a set of agents, where each agent only has access to its own local convex objective function, and the goal of the agents is to jointly minimize the sum of all the local functions. The communications among agents are described by a sequence of time-varying directed graphs which are assumed to be uniformly strongly connected. A column stochastic mixing matrices is employed in the algorithm which exactly steers all the agents to asymptotically converge to a global and consensual optimal solution even under the assumption that the step-sizes are uncoordinated. Two fairly standard conditions for achieving the geometrical convergence rate are established under the assumption that the objective functions are strong convexity and have Lipschitz continuous gradient. The theoretical analysis shows that the distributed algorithm is capable of driving the whole network to geometrically converge to an optimal solution of the convex optimization problem as long as the uncoordinated step-sizes do not exceed some upper bound. We also give an explicit analysis for the convergence rate of our algorithm through a different approach. Finally, simulation results illustrate the feasibility of the proposed algorithm and the effectiveness of the theoretical analysis throughout this paper.

1. Introduction

In recent years, with the rapid development of science and information technology, more and more researchers have paid their research attention to multi-agent systems and have obtained many remarkable achievements [7,8,13,16,19,21,23,25,38,43,47,52]. On the one hand, multi-agent system provides a theoretical research method for modeling and analysis of complex systems [18,40,41,42]. On the other hand, multi-agent system is also an important branch of distributed artificial intelligence research [5,15]. As one of the most important research subjects in the field of multi-agent systems, distributed optimization problem of multi-agent systems has attracted intensive research interest over the past few

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years due to their wide applications in distributed formation control of multiple autonomous vehicles [22], resource allocation in peer-to-peer communication networks [12], cloud computing [6,48,49], and distributed data fusion, information processing and decision making in wireless sensor networks [2,14,44], etc. Specifically, distributed optimization framework not only avoids establishing a long-distance communication system or data fusion center, but also provides better load balance to the network. In many networked systems, multi-agent systems are applied to solve such a distributed convex optimization problem where the goal is to minimize a sum of all the local objective functions, in which each local objective function is not known or shared by other agents.

In the existing literatures, distributed optimization problems are extensively solved by the (sub)gradient descent algorithm [1,17,24,34], the (sub)gradient-push descent algorithm [30], the fast (sub)gradient descent algorithm [10], and the dual averaging algorithm [4]. All of the above work [1,4,10,30,34] can be regarded as the consensus-based distributed (sub)gradient descent algorithms where each agent accomplishes a consensus step and then a descent step along the local (sub)gradient direction. During this time, many valuable consensus-based (sub)gradient algorithms have been addressed. Nedic and Ozdaglar [34] show that the consensus-based distributed (sub)gradient descent (DGD) algorithm converges at a rate of $O(1/\sqrt{t})$ for convex Lipshitz and possibly nonsmooth objective functions when applying a diminishing step size. This coincides with the convergence rate of the centralized subgradient descent algorithm. Then, Nedic and Olshevsky [30] propose a (sub)gradient-push descent algorithm, which drives every agent to an optimal value under a fairly standard assumption of (sub)gradient boundedness. Noting that the (sub)gradient-push descent algorithm in [30] requires no knowledge of the number of agents or the graph sequence to implement. Theoretical analysis demonstrated that the (sub)gradient-push algorithm converges at a rate of $O(\ln t/\sqrt{t})$ when using a diminishing step size. This line of work has been extended to a variety of realistic conditions for distributed optimization, such as directed [30] or random communication graph [28], stochastic (sub)gradient errors [46], heterogeneous local constraints [9,27], linear scaling in network size [39]. Although these algorithms are intuitive and simple, it cannot be neglected that they are usually slow. Among the reasons, even if the objective functions are differentiable and strongly convex, they still need to apply a diminishing step-size to converge to a consensus solution [4,9,10,11,26,27,28,29,30,32,37,39,45,51,53,54]. Also, the abovementioned algorithms all require the assumption of bounded (sub)gradient to achieve the exact optimal solution, which is another shortcoming. Furthermore, the abovementioned algorithms can be fast by using a fixed step-size, but they only converge to a neighborhood of the optimal solution set.

Recent work of Xu et al. [50] had coordinately put their sights on the analysis of a class of augmented distributed gradient method (Aug-DGM) of general linear time-invariant systems with fixed topology by applying the so-called Adapt–Then–Combine scheme. Specifically, this algorithm was proved to be convergent to an exact consensual minimizer for general convex and smooth objective functions when the uncoordinated step-size is sufficiently small. Then, Nedic et al. [32] studied the distributed optimization problems with coordinated step-size over time-varying undirected/directed graphs by combining the distributed inexact gradient method and the gradient tracking technique. The theoretical analysis showed that the distributed algorithm of Nedic et al. [32] was capable of driving the whole network to geometrically converge to an optimal solution under the assumption that the global objective function is strongly convex and has Lipschitz continuous gradient. Moreover, Nedic et al. in [33] also showed that the linear convergence rates could still be achieved in the studied algorithm for uncoordinated step-sizes, where the step-sizes do not exceed some upper bounds. It is worth noting that Nedic et al. [32] just discussed the distributed optimization problems with coordinated step-size over time-varying undirected/directed graphs, but they did not incorporates uncoordinated step-sizes with time-varying undirected/directed graphs. Furthermore, although the uncoordinated step-sizes was taken into account in the work of [33], it did not be extended to directed multi-agent systems in terms of the proposed framework.

Based on the above discussions, the main focus of this paper is to establish a distributed optimization algorithm in general directed multi-agent systems, which can guarantee a consensus and geometrically converge to the optimum under uncoordinated yet bounded step-sizes. We look forward to facilitating the development of a generalized theory of distributed optimization, and our ultimate goal is to design more realistic step-sizes which are capable of adaptability and promoting the practical applications. More precisely, the main contributions of this paper can be summarized as follows: (i) A distributed algorithm is proposed to analyze the convex optimization problem of multi-agent systems over time-varying directed graphs and yet uncoordinated step-sizes. (ii) It is important to note that, unlike the distributed descent method or the push-sum protocol proposed in [30,34], the proposed distributed algorithm is exactly ensured to converge to the minimizer even take account of applying uncoordinated step-sizes for local optimization. (iii) Theoretical analysis shows that the algorithm achieves a geometrical convergence rate as long as the uncoordinated step-sizes are smaller than an explicit upper bound and no positive lower bound is required when the objective functions are strong convexity and have Lipschitz continuous gradient. Specially, we construct linearly convergent methods along with establishment of explicit bounds on their convergence rates. (iv) Simulation results demonstrate that the algorithm has a faster convergence rates compared with the well-known distributed (sub)gradient descent (DGD) algorithm [34] and the push-sum algorithm [30].

The reminder of this paper is organized as follows. We begin in Section 2 where we present the notations, formulate the problem of interest, introduce the communication network, and give some useful assumptions. In Section 3, we consider the optimization algorithm along with the small gain theorem and some beneficial lemmas. The convergence and convergence rate results of the proposed distributed optimization algorithm are established in Section 4. Furthermore, the effectiveness of the algorithm is testified by applying a numerical example in Section 5. Finally, some conclusions and future directions are drawn in Section 6.