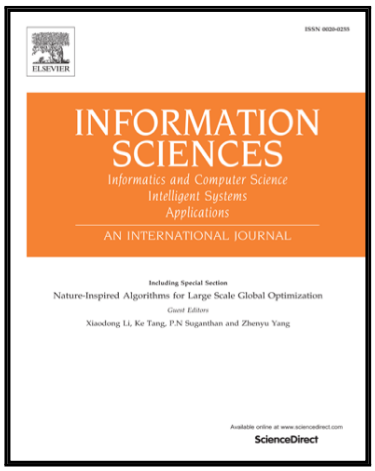

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Micro and Macro Models of Granular Computing induced by the Indiscernibility Relation.

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Abstract

In rough set theory (RST), and more generally in granular computing on information tables (GRC-IT), a central tool is the Pawlak's indiscernibility relation between objects of a universe set with respect to a fixed attribute subset. Let us observe that Pawlak's relation induces in a natural way an equivalence relation $\approx$ on the attribute power set that identifies two attribute subsets yielding the same indiscernibility partition. We call *indistinguishability relation* of a given information table $\mathcal{I}$ the equivalence relation $\approx$, that can be considered as a kind of global indiscernibility. In this paper we investigate the mathematical foundations of indistinguishability relation through the introduction of two new structures that are, respectively, a complete lattice and an abstract simplicial complex. We show that these structures can be studied at both a micro granular and a macro granular level and that are naturally related to the core and the reducts of $\mathcal{I}$. We first discuss the role of these structures in GrC-IT by providing some interpretations, then we prove several mathematical results concerning the fundamental properties of such structures.

1. Introduction

We denote by $\mathcal{I} := \langle U, Att, F, Val \rangle$ a knowledge representation system (information table in the finite case [42, 43, 44, 45]), having universe set U, attribute set Att, information map $F : U \times Att \to Val$ and value set Val. In this paper we introduce and study some micro-macro granular mathematical structures uniquely associated to any knowledge representation system $\mathcal{I}$.

1.1. General Premise

Tabular representation of data appears in several fields of research, related to many problems of taxonomy in biology, economics, social sciences and so forth. At present, many researchers are dedicating themselves to the analysis of Pawlak's information tables only through heuristic interpretations and by using an informatic vocabulary. We mainly aim to introduce and investigate Pawlak's indiscernibility relation between attributes instead of the usual relation between objects. In order to distinguish the two relations, the term *indistinguisuishability* will be used in case of attributes. That is, the indistinguishability is simply the equivalence relation $\approx$ on the power set $\mathcal{P}(Att)$ that identifies two attribute subsets inducing on U the same indiscernibility relation. The surprising fact is that the indistinguishability relation induces a very rich interrelation of several mathematical structures, that we will call *indiscernibility hypergraphic structures*, and that we will study to different granularity levels. In this paper, we are mainly interested in developing the mathematical foundations relying on a series of old and new hypergraphic structures induced by the Pawlak's indiscernibility relation on information tables. We study the basic mathematical properties of such structures that arise in a natural way without assuming any extra hypothesis. One of our basic motivations is to understand which kind of formal theory can be developed in terms of order structures, hypergraphic structures and their potential links arising when we investigate an information table with GrC methodologies (abbreviated GrC-IT). From this standpoint, we think that a granular

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