Neurocomputing 207 (2016) 408–415

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Observer-based robust consensus control for multi-agent systems with noises

Zhongmei Wang, Huanshui Zhang *

School of Control Science and Engineering, Shandong University, Jinan, Shandong 250061, China

ARTICLE INFO

Article history:
Received 5 November 2015
Received in revised form
3 May 2016
Accepted 6 May 2016
Communicated by Rongni Yang
Available online 16 May 2016

Keywords:
Multi-agent systems
Consensus error
System noise
Measurement noise
Communication noise
Kalman filter

ABSTRACT

This study deals with the problem of consensus error calculation for linear multi-agent systems (MASs) with system noises and measurement noises. It is supposed that there exist communication noises when the information exchanges among distinct agents. The agents cannot reach consensus due to the existence of the system noises. We aim to design proper distributed control protocol to calculate the consensus error for the MASs. The states of each agent and its neighbors are estimated by applying Kalman-filtering theory. The distributed control protocol is designed by employing the estimated state information. Then the consensus error is calculated via the unique solution of a Lyapunov equation. Finally, a simulation example is presented to verify the correctness of the proposed results.

1. Introduction

The model of multi-agent systems (MASs) has attracted great interest in the last decades due to its wide applications in the research of sensor networks [1–3], neural networks [4,5], and vehicle formations [6,7]. As one of the fundamental research topics arising from cooperative control for multi-agent systems, consensus control has received considerable attention in various scientific fields ranging from mathematics to control engineering. For the multi-agent systems, each agent can only acquire local information about itself and its neighbors, and the objective of consensus is to design distributed control protocol by using local information such that all the agents reach to an agreement on a common value for some interest. Plentiful results have been reported in literature for various multi-agent systems, such as for the MASs with time delay [8–10], with limited communication data rate [11], with nonlinear dynamics [12–14], and so on.

Recently, the consensus problem for the MASs with communication noises has become an attractive topic. The heart of the matter is to attenuate the effect of communication noises. In [15], a time-varying gain, namely the stochastic-approximation type gain, was first introduced in the consensus protocol. In [16], the stochastic-approximation type gain was proved to be necessary and sufficient for ensuring the mean square average consensus of the first-order integral MASs. In [17], both cases of communication noises and time delays were considered, and it is proved that the stochastic-approximation type consensus protocol is still valid in this situation. The consensus problems of second-order and general linear dynamics were studied in [18,19], respectively, and it is proved that the stochastic-approximation type conditions are still necessary and sufficient for mean square average consensus.

However, stochastic system noises are unavoidable to affect the dynamics of the agents in real world. The consensus problem for the MASs with system noises is considered in [20,21]. In [20], a robust consensus condition was given for a special class of MASs motivated by the Vicsek model. In [21], the consensus convergence for first-order, continuous-time multi-agent systems with input noises was established. To the best of our knowledge, the consensus problem for the multi-agent systems with both system noises and communication noises has not yet attracted adequate research attention.

Motivated by the above discussion, this paper focuses on the observer-based distributed robust consensus control for the general linear MASs with both system noises and communication noises. It is assumed that there are N agents in the multi-agent systems. The dynamics of each agent is a continuous-time control system with system noises. Suppose that the output of the agent is available via a noisy measurement equation. Moreover, it is supposed that each agent receives information from its neighbors via noisy communication channel. It is well known that the MASs cannot reach consensus due to the existence of the system noises.

* Corresponding author.
E-mail addresses: wangzhongmei211@gmail.com (Z. Wang), hszhang@sdu.edu.cn (H. Zhang).

http://dx.doi.org/10.1016/j.neucom.2016.05.029
0925-2312/

Our aim is to design distributed control protocol to calculate the consensus error for the multi-agent systems.

The rest of this paper is organized as follows. Section 2 contains some preliminary results for graph theory, some basic facts about consensusability for MASs without noises, and the problem formulation of this paper. In Section 3, a distributed protocol is proposed based on the estimated state information of the agent and its neighbors, and the consensus error of the multi-agent systems is obtained via the solution of a Lyapunov equation. In Section 4, a simulation example is presented. Section 5 is a brief conclusion.

2. Preliminaries and problem formulation

2.1. Notations and graph theory

$\mathbb{R}^{m\times n}$ denotes the set of $m\times n$ real matrices; $1_n=(1,1,\dots,1)^T\in\mathbb{R}^n$; I_n denotes the identity matrix with dimension n. For a vector $x\in\mathbb{R}^n$, $\|x\|$ denotes its 2-norm. For a matrix $A\in\mathbb{R}^{n\times n}$, A^T denotes its transpose;tr(A) denotes its trace; $\lambda_i(A)$, $i=1,2,\dots,n$, denote its eigenvalues; Re $\{\lambda_i(A)\}$ denotes the real part of $\lambda_i(A)$. For a sequence of vectors $\alpha_1,\alpha_2,\dots,\alpha_N$, denotes col $[\alpha_1,\alpha_2,\dots,\alpha_N]=[\alpha_1^T,\alpha_2^T,\dots,\alpha_N^T]^T$. $A\otimes B$ denotes the Kronecker product of matrices A and B. $E(\nu)$ denotes the mathematical expectation of the random variable ν. The notation $\delta_{(t-\tau)}$ denotes the delta function, i.e., $\delta_{(t-\tau)}=1$ if and only if $t=\tau$, while $\delta_{(t-\tau)}=0$ if and only if $t\neq\tau$.

In this paper, the communication topology of the MASs is formulated by an undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$, where $\mathcal{V}=\{1,2,\dots,N\}$ is a nonempty finite set of nodes and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is a set of edges. An edge $(i,j)\in\mathcal{E}$ means that nodes i and j communicate with each other. An undirected graph is connected if there exists at least one path between any pair of nodes. The set of all the neighbors of node i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}|(j,i)\in\mathcal{E}\}$. $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is the adjacency matrix of $\mathcal{G}$. $a_{ij}>0$ if and only if $(j,i)\in\mathcal{E}$. Moreover, we assume that $a_{ii}=0$, for any $i\in\mathcal{V}$. It is noted that for an undirected graph $\mathcal{G}$, $\mathcal{A}$ is a symmetric matrix. The degree matrix $\mathcal{D}=\mathrm{diag}\{d_1,d_2,\dots,d_n\}\in\mathbb{R}^{N\times N}$ is a diagonal matrix with $d_i=\sum_{j=1}^N a_{ij}$, $i=1,2,\dots,N$. The Laplacian matrix $L_\mathcal{G}=[l_{ij}]_{N\times N}$ is given by $L_\mathcal{G}=\mathcal{D}-\mathcal{A}$. Note that $l_{ij}=-a_{ij}$ for $i\neq j$, and $l_{ii}=d_i$ for $i=1,2,\dots,N$.

Lemma 1 (*Godsil and Royle [22], Ren and Beard [23]*). *Let $\mathcal{G}$ be an undirected graph, its Laplacian matrix $L_\mathcal{G}$ is symmetric and for all $i\in\mathcal{V}$, $\lambda_i(L_\mathcal{G})\ge 0$. Moreover, $\mathcal{G}$ is connected if and only if $L_\mathcal{G}$ has exactly one zero eigenvalue, and all the eigenvalues of $L_\mathcal{G}$ can be written in an ascending order as $0=\lambda_1(L_\mathcal{G})<\lambda_2(L_\mathcal{G})\le\dots\le\lambda_N(L_\mathcal{G})$.*

2.2. Preliminaries on the consensusability for the MASs without noises

In this subsection, we recall some basic facts with respect to the consensusability of the MASs without noises. The dynamics of the ith agent is described as

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\quad i=1,2,\dots,N. \tag{1}$$

Assume that (A,B) is stabilizable and the undirected communication graph $\mathcal{G}$ is connected. The definition of consensusability is first introduced in [24] as follows.

Definition 1 (*Ma and Zhang [24]*). For the system (1), if there exists a distributed protocol $\mathcal{U}$ such that

$$\lim_{t\to\infty}\|x_j(t)-x_i(t)\|=0,\quad i,j=1,2,\dots,N, \tag{2}$$

for any initial value $x_i(0)$, then we say that MASs (1) is consensusable with respect to $\mathcal{U}$.

From [25], it is noted that if (A,B) is stabilizable, the following Riccati equation

$$A^TZ+ZA-ZBB^TZ+I_n=0 \tag{3}$$

has a unique nonnegative definite solution Z.

According to the results of [24], the following distributed protocol

$$u_i(t)=K\sum_{j\in\mathcal{N}_i}a_{ij}(x_j(t)-x_i(t)) \tag{4}$$

can ensure (1) reach consensus, where $K=\max\{1,\epsilon_0^{-1}\}BB^TZ$, $\epsilon_0=\lambda_2(L_\mathcal{G})$, and Z is the unique nonnegative definite solution of (3).

Under the assumption that (A,B) is stabilizable and $\mathcal{G}$ is connected, we note that the MASs (1) can reach consensus with respect to (4) if and only if the following error system

$$\dot{\tilde{\delta}}(t)=(\overline{A}-\overline{B}\,\overline{K}^{(1)})\tilde{\delta}(t) \tag{5}$$

is stable, which equals to that $\overline{A}-\overline{B}\,\overline{K}^{(1)}$ is a Hurwitz matrix, where $\tilde{\delta}(t)=\mathrm{col}[x_2(t)-x_1(t),x_3(t)-x_1(t),\dots,x_N(t)-x_1(t)]$ is the global state error vector; $\overline{A}=I_{N-1}\otimes A$, $\overline{B}=I_{N-1}\otimes B$; $\overline{K}^{(1)}=(L_1+1_{N-1}\cdot\alpha_1^T)\otimes K$ with

$$L_1=\begin{bmatrix} d_2 & -a_{23} & \cdots & -a_{2N}\\ -a_{32} & d_3 & \cdots & -a_{3N}\\ \vdots & \vdots & & \vdots\\ -a_{N2} & -a_{N3} & \cdots & d_N\end{bmatrix}$$

and $\alpha_1=[a_{12},a_{13},\dots,a_{1N}]^T$. Here, we would like to point out that throughout the paper the notations a_{ij} $(i,j\in\mathcal{V})$ always denote the adjacency elements as defined above.

2.3. Problem formulation

In this paper, we consider MASs composed of N agents with undirected communication topology $\mathcal{G}$. The dynamics of the ith agent is described by the following linear time-invariant system:

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t)+w_i(t), \tag{6}$$

$$y_i(t)=Cx_i(t)+v_i(t), \tag{7}$$

where $x_i(t)\in\mathbb{R}^n$, $u_i(t)\in\mathbb{R}^m$, and $y_i(t)\in\mathbb{R}^r$ denote, respectively, the state, the control input, and the output of the ith agent; A and B are constant known matrices with compatible dimensions; the initial state of the ith agent is denoted by $x_i(0)$ with mathematical expectation and variance being χ_i and $\Sigma_i(0)$, respectively; $\{w_i(t):i\in\mathcal{V}\}$ and $\{v_i(t):i\in\mathcal{V}\}$ are zero-mean white noises, and have covariance $Q\delta_{(t-\tau)}$ and $R\delta_{(t-\tau)}$, respectively; $Q\ge 0$ and $R>0$.

The neighbors' interfered state information measured by agent i is described as

$$y_j^{(i)}(t)=Cx_j(t)+\theta_j^{(i)}(t),\quad j\in\mathcal{N}_i, \tag{8}$$

where the communication noise $\theta_j^{(i)}(t)$ is assumed to be a zero-mean white noise process with covariance $R\delta_{(t-\tau)}$. In this paper, it is assumed that all the noises and stochastic variables are mutually independent. The following assumptions are required in this paper.