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ORIGINAL ARTICLE

Existence of Solution of Nonlinear Fuzzy Fredholm Integro-differential Equations

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Abstract In this paper, we prove some results concerning the existence of solution of a class of nonlinear fuzzy Fredholm integro-differential equations. Also an iterative approach is proposed to obtain approximate solution of a class of nonlinear fuzzy Fredholm integro-differential equation of the second kind. A numerical example is presented to illustrate the proposed method.

Keywords Fuzzy numbers · Fuzzy integral · Nonlinear fuzzy integro differential equations · Numerical methods

1. Introduction

The topics of fuzzy integral equations (FIE) and fuzzy differential equations have been rapidly growing in recent years [1-13]. The fuzzy mapping function was introduced by Chang and Zadeh [14]. Later, Dubois and Prade [15] presented an elementary fuzzy calculus based on the extension principle. Also the concept of integration of fuzzy functions was first introduced by them. Then the fuzzy integration is discussed by Allahviranloo [16], Allahviranloo and Otadi [17, 18] and Mosleh and Otadi [19].

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In the existence of the solution of fuzzy integral equations, the Ascoli's theorem or metric fixed point theorems are used. For the existence and uniqueness, the main tool is the Banach fixed point principle. Such discussions can be found in [20-24].

Babolian et al. and Abbasbandy et al. [25, 26] obtained a numerical solution of linear Fredholm fuzzy integral equations of the second kind. Then Otadi and Mosleh [27] considered fuzzy nonlinear integral equations of the second kind and obtained an approximate solution to the fuzzy nonlinear integral equations. In [23] the author considered nonlinear fuzzy Fredholm integral equations such as

$$F(s) = f(s) \oplus \int_a^b K(s,t,F(t))dt,$$

therefore, in this paper, we generalize the nonlinear fuzzy integral equations to the nonlinear fuzzy integro-differential equations

$$F'(s) = f(s) \oplus \int_a^b K(s,t,F(t))dt, \ F(a) = F_0.$$

In this paper, we present a simple numerical method to nonlinear fuzzy Fredholm integro-differential equations of the second kind.

2. Preliminaries

In this section, the basic notations used in fuzzy operations are introduced. We start by defining the fuzzy number.

Definition 2.1 *A fuzzy number is a function $u : \mathbb{R} \longrightarrow I = [0,1]$ having the properties* [28]*:*

(i) *u is normal, that is $\exists x_0 \in \mathbb{R}$ such that $u(x_0) = 1$;*

(ii) *u is a fuzzy convex set;*

(iii) *u is upper semicontinuous on $\mathbb{R}$;*

(iv) *The support $\overline{\{x \in \mathbb{R} \mid u(x) > 0\}}$ is a compact set.*

The set of all the fuzzy numbers is denoted by E. An alternative definition which yields the same E is given by Kaleva [29].

Definition 2.2 *A fuzzy number u is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r)$ and $\overline{u}(r)$, $0 \leq r \leq 1$, which satisfies the following requirements* [29]*:*

(i) *$\underline{u}(r)$ is a bounded monotonically non-decreasing, left continuous function on $(0,1]$ and right continuous at 0;*

(ii) *$\overline{u}(r)$ is a bounded monotonically non-increasing, left continuous function on $(0,1]$ and right continuous at 0;*

(iii) $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

A crisp number r is simply represented by $\underline{u}(\alpha) = \overline{u}(\alpha) = r, 0 \leq \alpha \leq 1$. This fuzzy number space as shown in [30], can be embedded into the Banach space $B = \overline{C}[0,1] \times \overline{C}[0,1]$.

For arbitrary $u = (\underline{u}(r), \overline{u}(r)), v = (\underline{v}(r), \overline{v}(r))$ and $k \in \mathbb{R}$, we define addition and multiplication by k as