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Some Special Sequences in Fuzzy Graphs

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Abstract In a fuzzy graph, the edges are mainly classified into α, β and δ. In this paper, some sequences in fuzzy graphs are introduced, whose concepts are based on the classification of edges. Besides, characterizations for blocks in fuzzy graphs and fuzzy trees are obtained. It is shown that β sequence of a fuzzy tree is a zero sequence and α sequence of a block is a binary sequence.

Keywords α-Sequence · β-Sequence · Strong sequence · Block · Fuzzy tree

1. Introduction and Preliminaries

Graph theory has now become a major branch of applied mathematics due to its large variety of applications and effectiveness. Graph is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. In models, when we have an uncertainty about either the set of vertices or the set of edges or both, the model becomes a fuzzy graph. Currently, the theory of fuzzy graphs is an intense area of research. Fuzzy graphs differ from the classical ones in several ways, among them the most prominent one is connectivity. Distance and central concepts also play important roles in applications related with fuzzy graphs.

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Rosenfeld [15] gave a mathematical definition for a fuzzy graph in 1975. Several authors including Bhutani [2-5] and Sunil Mathew and Sunitha [18, 19, 21-24] introduced many connectivity concepts in fuzzy graphs following the jobs of Zadeh and Rosenfeld [6, 15, 26, 27]. More related researches can be seen in [1, 6-11, 13, 14, 16, 17, 25].

In this article, we introduce three new sequences in fuzzy graphs. These concepts are derived from the notion of connectivity in fuzzy graphs. In a fuzzy graph model, for example, in an information network or an electric circuit, the reduction of flow between pairs of nodes is more relevant and may frequently occur than the total disruption of the entire network [7, 8, 11]. Finding the center of a graph is useful in facility location problems where the goal is to minimize the distance to the facility. For example, placing a hospital at a central point reduces the longest distance the ambulance has to travel. This concept represents our motivation. As fuzzy graphs are generalized structures of graphs, the concepts introduced in this article also generalize the classic ideas in graph theory.

A fuzzy graph (f-graph for short) [15] is a pair $G : (\sigma, \mu)$ where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric. In all the examples, σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \epsilon S \mid \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \epsilon SXS \mid \mu(u, v) > 0\}$. An f-graph $H : (\tau, \nu)$ is called a partial fuzzy subgraph of $G : (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for every u and $\nu(u, v) \leq \mu(u, v)$ for every pair of nodes u and σ^*. In particular, we call $H : (\tau, \nu)$ a fuzzy subgraph of $G^* : (\sigma^*, \mu^*)$ if $\tau(u) = \sigma(u)$ for every $u \epsilon \tau^*$ and $\nu(u, v) = \mu(u, v)$ for every $(u, v) \epsilon \nu^*$. A path P of length n is a sequence of distinct nodes $u_0, u_1, \cdots u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, \cdots, n$ and the degree of membership of a weakest edge is defined as its strength. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$. An $x - y$ path P is called a strongest $x - y$ path if its strength equals $CONN_G(x, y)$ [12]. An f-graph $G : (\sigma, \mu)$ is connected if for every x, y in $\sigma^*, CONN_G(x, y) > 0$. Through out, we assume that G is connected. An edge (x, y) of an f-graph is called strong if its weight is at least as great as the connectedness of its end nodes x and y in the edge deleted fuzzy subgraph $G - (x, y)$ and an $x - y$ path P is called a strong path if P contains only strong edges [5].

An edge in an f-graph G is called a fuzzy bridge (f-bridge for short) of G if its removal reduces the strength of connectedness between some pair of nodes in G [12]. Similarly, a fuzzy cut node (f-cut node for short) w is a node in G whose removal from G reduces the strength of connectedness between some pair of nodes other than w. An f-graph having no fuzzy cut nodes is called a block [12].

2. Sequences in an f-Graph

The problem of determining the structure of an f-graph is really a challenging one. In [18], Mathew and Sunitha provided an algorithm for the identification of different types of edges in an f-graph. This algorithm is very effective even for fuzzy graphs with a large number of vertices. Based on this categorization, we introduce certain sequences, which will, very effectively determine the nature and structure of certain