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Modal decoupling of overhead transmission lines using real and constant matrices: Influence of the line length

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ABSTRACT

The Clarke's matrix is a well-known real and constant transformation matrix used for modal transformation in three-phase transmission lines modeling. Although modal analysis has been widely discussed in the technical literature on power system modeling, a new content is approached in this research proving that the approximation using an exact and constant modal transformation matrix depends on both the frequency-dependent parameters and transmission line's length. As an important conclusion, the approach using the Clarke's matrix leads to more accurate results considering long transmission lines. There are two methods for modal decoupling in power systems modeling. The first uses only a single constant and real transformation matrix during the entire modeling/simulation routine. The second uses the frequency-dependent transformation matrix for parameters decoupling into the propagation modes and the Clarke's matrix for mode-to-phase transformation of voltage and current values during simulations. The accuracy of these two modeling/simulation processes are evaluated, in the time and frequency domains, based on results obtained from a reference routine that employs the exact frequency-dependent matrix in modal transformations and numerical transforms for simulation in the time domain. The proposed analysis proves that the accuracy of both methods varies with the line length during electromagnetic transient simulations that leads to peak errors up to approximately 10%. The influence of the line length in modal analysis techniques was not approached in previous references on power system modeling, which represents the original contribution of this paper.

1. Introduction

Modal analysis techniques are widely applied in power systems modeling [1–4]. In this context, a coupled multiphase system can be decoupled into propagation modes that can be modeled separately as individual single-phase systems. Modal transformations are successively applied during the modeling and simulation routines to convert line parameters, voltages and currents from the phase domain to modal domain and vice versa [5]. These transformations are carried out by using modal transformation matrices, which are usually frequency-dependent due to frequency effect on the line parameters. However, depending on the line geometry and system characteristics, several approximations can be achieved in order to produce constant modal transformation matrices, e.g.: symmetrical components, Karrenbauer, Clarke and others. The modal decoupling theory represents an essential tool in power systems modeling for analysis of insulation coordination, electromagnetic compatibility, protection and general design of transmission lines.

Transmission line models for simulation of electromagnetic transients are usually presented in the technical literature into two categories: distributed- or lumped-parameters models. The first is developed directly from the frequency-dependent distributed parameters of the line and using modal decoupling, where each propagation mode is represented as an independent two-port circuit and simulation results are obtained in the time domain from numerical transforms [6]. The second category is also based on modal decoupling, where each propagation mode is represented as a single-phase line by electric circuit approach and the frequency effect on the line parameters is included directly in the time domain by means of fitting techniques [3,4,7].

The two modeling techniques present advantages and restrictions that were well established in the literature on transmission line modeling [1,5]. However, some important issues should be emphasized for an appropriate understanding of the proposed analysis. Although the distributed-parameters models show a great

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accuracy for a wide range of frequencies, *i.e.*, for any of electromagnetic transient in power systems, since a switching up to a very fast steep-front impulse (atmospheric impulse), these frequency-domain models have several limitations for inclusion of non-linear and time-varying events during simulations [8,9]. On the other hand, lumped-parameters models are versatile in the inclusion of time-varying elements during time-domain simulations (e.g. corona effect, non-linear loads and fault occurrences) [3,4]. However, the multiconductor line modeling by lumped elements presents some numerical errors due to the numerical solution of the differential equations and requires a real and constant modal transformation matrix for voltage and current transformation from the modal domain to the phase domain. The use of a real and constant matrix instead the frequency-dependent modal matrix, which is calculated directly from the admittance and impedance matrices, represents a valid approximation only for transmission lines with vertical symmetry plane. Thus, some errors are expected from such approximation, which were also evaluated in the technical literature [2]. Alternative techniques were also proposed for reduction of these errors, based on the alternation in the use of the exact and Clarke's matrices during the modeling and simulation process [5].

In this context, an additional and original analysis is proposed for evaluation of possible errors in the transmission line modeling, using modal techniques, as a function of the line length. Eventual errors in the frequency domain are further analyzed in the time domain by means of electromagnetic transient simulations, *i.e.*, in terms of wave shape, voltage and current peaks which power systems are subject during the occurrence of a steep-front impulse.

2. Three-phase line models using modal analysis

The modal decoupling consists into decouple a three-phase transmission line into three independent propagation modes, which can be represented as three single-phase lines. In this context, the differential equations of a multiconductor line are introduced as follows [2]:

$$\frac{d[V_{ph}]}{dx} = -[Z][I_{ph}] \tag{1}$$

$$\frac{d[I_{ph}]}{dx} = -[Y][V_{ph}] \tag{2}$$

Terms $[Z]$ and $[Y]$ are the impedance and admittance matrices of the line, respectively. The phase voltages and currents are in vectors $[V_{ph}]$ and $[I_{ph}]$, respectively. The solution of Eqs. (1) and (2) is not a trivial procedure because the impedance and admittance matrices have mutual terms, *i.e.*, phases of the multiconductor line are coupled by mutual terms.

By differentiating (1) and (2) and substituting the first derivatives back into the second derivatives, the following expressions are obtained:

$$\frac{d^2[V_{ph}]}{dx^2} = [Z][Y][V_{ph}] = [S_V][V_{ph}] \tag{3}$$

$$\frac{d^2[I_{ph}]}{dx^2} = [Y][Z][I_{ph}] = [S_I][I_{ph}] \tag{4}$$

Since $[Z]$ and $[Y]$ are symmetrical, the product $[Z][Y]$ and $[Y][Z]$, in Eqs. (3) and (4), respectively, are defined as $[S_V]$ and $[S_I]$ that are also transposed each other:

$$[S_V] = [S_i]' \tag{5}$$

However, $[S_V]$ and $[S_I]$ are not symmetrical.

Due to relationship (5), $[S_V]$ and $[S_I]$ share the same polynomial characteristic and consequently have the same eigenvalues $[\lambda]$. Nonetheless, a matrix and its transpose do not have the same eigenvectors. Thus, the matrix with eigenvalues $[\lambda]$ is related to $[S_V]$ and $[S_I]$ through the eigenvectors $[T_V]$ and $[T_I]$:

$$[\lambda] = [T_V]^{-1}[S_V][T_V] = [T_V]^{-1}[Z][Y][T_V] \tag{6}$$

$$[\lambda] = [T_I]^{-1}[S_I][T_I] = [T_I]^{-1}[Y][Z][T_I] \tag{7}$$

Isolating the products $[Z][Y]$ and $[Y][Z]$ from (6) and (7) and substituting them in (3) and (4), the following expressions are obtained:

$$\frac{d^2[T_V]^{-1}[V_{ph}]}{dx^2} = [\lambda][T_V]^{-1}[V_{ph}] \equiv \frac{d^2[V_m]}{dx^2} = [\lambda][V_m] \tag{8}$$

$$\frac{d^2[T_I]^{-1}[I_{ph}]}{dx^2} = [\lambda][T_I]^{-1}[I_{ph}] \equiv \frac{d^2[I_m]}{dx^2} = [\lambda][I_m] \tag{9}$$

From Eqs. (8) and (9), the voltages and currents in the modal domain are identified and can be expressed as:

$$[V_m] = [T_V]^{-1}[V_{ph}] \tag{10}$$

$$[I_m] = [T_I]^{-1}[I_{ph}] \tag{11}$$

Defining Eqs. (10) and (11) from (1) and (2):

$$\frac{d[V_m]}{dx} = -[T_V]^{-1}[Z][T_I][I_m] \tag{12}$$

$$\frac{d[I_m]}{dx} = -[T_I]^{-1}[Y][T_V][V_m] \tag{13}$$

The modal impedance matrix $[Z_m]$ and modal admittance matrix $[Y_m]$ are defined:

$$[Z_m] = [T_V]^{-1}[Z][T_I] \tag{14}$$

$$[Y_m] = [T_I]^{-1}[Y][T_V] \tag{15}$$

The transformation matrices $[T_I]$ and $[T_V]$ in Eqs. (10)–(15) vary with the frequency because $[Y]$ and $[Z]$ are also frequency dependent. The relationship of the transformation matrices is expressed [4]:

$$[T_V]^{-1} = [T_I]^T \tag{16}$$

The modal matrices $[Z_m]$ and $[Y_m]$ are diagonal and are calculated as a function of the frequency. Since the modal matrices are diagonal, each propagation mode is completely decoupled from each other and can be represented as a single-phase transmission line. This way, the phase-mode-phase conversion during modeling and simulation routines can be described in Fig. 1.

3. Single-phase line representation

As described in the previous section, the solution of multiconductor line Eqs. (1) and (2) is possible from the line decoupling into n independent propagation modes. This way, each mode can be modeled as a single-phase line using several techniques based on the representation by distributed parameters in the frequency domain or by lumped parameters in the time domain. As the goal of the proposed analysis is to evaluate the accuracy in the use of modal techniques as a function of the line length, the line representation by two-port circuit is the most accurate method for modeling the propagation modes without errors in the electrical parameters representation [2,5]. The frequency-domain equations