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A powerful method for solving the power flow problem in the ill-conditioned systems

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ABSTRACT

In most cases, the power systems are well-conditioned and the power flow problem (PFP) can be solved by using the famous Newton or Newton-based methods. However, in some cases, the conditions of the power systems are ill and the above-mentioned methods are poorly converged or even diverged. This paper presents application of corrected Levenberg-Marquardt algorithm with a non-monotone line search for solving the PFP in the ill-conditioned power systems. The presented algorithm is evaluated on the case studies ranging from small to large (30-bus, 57-bus, 118-bus and 2383-bus). Simulation results show the proposed approach converges in all of the case studies. Moreover, application of the proposed method for solving the PFP in ill-conditioned power systems can significantly reduce the CPU time and number of iterations in comparison with the benchmark methods.

1. Introduction

1.1. Previous works

Solving the power flow problem (PFP) is one of the fundamental issues used in the steady state analysis of power systems. Using digital computers to solve PFP started in mid 1950s [1]. Since then, different algorithms have been used in power flow (PF) calculations. The developments of these algorithms are mainly compared by the basic requirements of the PFP calculations. These requirements and indices are as follows:

- The convergence characteristics
- The computing efficiency (CPU time) and memory requirements
- The flexibility and reliability

One of the classical techniques for solving the PFP is based on Gauss–Seidel technique (GST). The number of the iterations in GST is high and its convergence characteristic is poor [2]. The most famous and popular technique for solving the PFP is Newton's technique (NT). In 1956, the studies for developing the PFP calculations were started by Ward and Hale [3]. The effective starting processes for using Newton's method in the PFP are presented in [4–7]. Many literatures have been based on the traditional NT. Some of these methods are decoupled technique, fast-decoupled technique (FDT) and the second order Newton's method [8–11]. Ref. [12] presents a robust FDT for solving the PFP that is suitable for systems with high r/x ratio lines.

Authors of [13] have presented a second order PF method using equations of current injection instead of classical rectangular PF equations. Ref. [14] presents an iterative technique for PFP to improve the computation complexity (CPU time and number of iterations). In this technique, impedance matrix (Z_{bus}) has been used instead of admittance matrix (Y_{bus}).

Authors have recently published a paper on using high-order Newton-like methods for solving the PF equations [15]. The main capabilities of these methods are having simple structures, being faster than the traditional NT and significantly reducing the CPU time.

The main objectives of solving the PFP are to determine the voltage magnitude and voltage phase, as well as reactive and real powers of each bus in a power system.

By using the Newton's method or Newton-like methods [15] during the solution of the PFP, the Jacobian matrix may become non-singular, near singular or singular:

- Non-singular Jacobian matrix ($|\boldsymbol{J}| \neq \mathbf{0}$)

In this case, the PFP solution exists and is obtainable using a flat initial guess. When the Jacobian matrix of a power system is non-singular, the system is named well-conditioned and the PFP can be solved by the traditional NT and other Newton-like methods [15]. In this situation, the number of iterations in the PFP solution is small.

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- Near singular or singular Jacobian matrix ($|\boldsymbol{J}| \sim \boldsymbol{0}$ or $|\boldsymbol{J}| = \boldsymbol{0}$)

In these situations, the PFP solution exists, but using the traditional GST, NT and Newton-like methods for solving the PFP will cause slow convergence or even divergence of the solution. The power system with near singular or singular Jacobian matrix is named bad-conditioned or ill-conditioned [16,17].

Some reasons may lead to change the condition of the power system to ill condition. Some of these reasons are position of the swing bus, installation of some equipment such as flexible AC transmission system (FACTS) and high ratio of r/x in radial networks. The solution of the PFP in the ill-conditioned power systems is very sensitive to small changes in the parameters of the Jacobian matrix [18,19].

The Iwamoto method is one of the vastly applied techniques being used for solving the PFP in ill-conditioned power systems [20]. The main idea behind the Iwamoto method is to find the optimal multiplier parameter to minimize the power residuals ($\Delta\boldsymbol{P}$ and $\Delta\boldsymbol{Q}$). Many other literatures have focused on solution of the PFP in the ill-conditioned power systems [21–27]. Ref. [28] has used the concavity theory to solve an optimal multiplier PFP with low voltage solutions. In this method, polar coordinate system has been replaced instead of the rectangular coordinate one that can simplify the task further. The replaced polar system has applied the second order power flow equations for reducing the CPU time. Authors in [29] have used the quadratic discriminant index to enhance optimal multiplier power flow technique for finding low voltage solutions at the maximum loading point.

The Levenberg-Marquardt (LM) technique presented in [30] is a reliable and efficient method for solving the PFP in ill-conditioned systems. Ref. [18] has presented a continuous version of the Newton's algorithm for solving the PFP in ill-conditioned power systems using a set of autonomous ordinary differential equations (ODE). Moreover, some methods based on NT and LM method have been presented in [31,32].

We have recently presented a high-order Levenberg-Marquardt method to solve the PFP in ill-conditioned power systems [33] and shown that by using the controlling parameters of this method, the number of iterations and CPU time can be decreased. In the previous work of authors [33], the rate of convergence is four but in the presented method, the rate of convergence is global.

1.2. Contribution & paper organization

The main contribution of this paper is to present an algorithm for solving the PFP in ill-conditioned power systems that can reduce the calculation time and number of iterations. The proposed approach is based on a mathematical method which is termed corrected Levenberg–Marquardt technique with a non-monotone line search [34] but application of this mathematical approach in a practical power system needs some requirements as follows:

(a) Modification of the method based on the structure and characteristics of PFP in the power systems.
(b) Presentation of a proper algorithm to apply the modified formulation to solve the PFP in the power systems.

It will be shown that the proposed method converges in all of ill-conditioned power systems with high condition numbers.

This paper is organized as follows: Section 2 presents the concepts of well- and ill-conditioned power systems based on the condition numbers of systems. The mathematical formulation of the method is introduced in Section 3. In Section 4, the proposed algorithm for solving the PFP in ill-conditioned systems is presented. The case studies and their condition numbers are listed in Section 5. Simulation results of the proposed approach are reported and compared with some benchmark methods in Section 6. Finally, concluding remarks are presented in Section 7.

2. Ill- and well-conditioned systems

Consider a system with the set of simultaneous equations:

$$\sum_{l=1}^{n} j_{il} x_l = v_i \quad i = 1, 2, \ldots n \tag{1}$$

That can be written in the matrix form as follows:

$$\boldsymbol{J}\boldsymbol{X} = \boldsymbol{V} \tag{2}$$

In the equations of this system, $\boldsymbol{J}$ ($[j_{il}]$) is the coefficient matrix and $\boldsymbol{X}$ and $\boldsymbol{V}$ are column matrices. There are some indices for recognition of the ill-conditioned systems from the well-conditioned one. The most famous index in this area is the condition number (CN) that is presented by Von Neumann and Goldstine [16]:

$$CN = \frac{\xi_{max}}{\xi_{min}} \tag{3}$$

where ξ_{min} and ξ_{max} are the smallest and largest eigenvalues of the coefficient matrix, respectively. As the condition number increases, the degree of the ill-conditioned system grows as well [15,21].

3. Presentation of non-monotone line search with corrected Levenberg–Marquardt (NLS-CLM) technique

3.1. Background

Consider the system of algebraic nonlinear equations:

$$\boldsymbol{F}(\boldsymbol{x}) = 0 \tag{4}$$

The Levenberg–Marquardt (LM) method is a conventional technique for solving the algebraic nonlinear equations of an ill-conditioned system. At each iteration:

$$\boldsymbol{\phi}_k^{LM} = -(\boldsymbol{J}_k^T\boldsymbol{J}_k + \lambda_k\boldsymbol{I})^{-1}\boldsymbol{J}_k^T\boldsymbol{F}_k \tag{5}$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{\phi}_k^{LM} \tag{6}$$

where λ_k is nonnegative parameter, $\boldsymbol{F}_k = \boldsymbol{F}(\boldsymbol{x}_k)$, $\boldsymbol{J}_k$ is Jacobian matrix of $\boldsymbol{F}_k$ at $\boldsymbol{x}_k$. Note that to dominate the difficulty in case of singularity or near singularity of $\boldsymbol{J}_k^T\boldsymbol{J}_k$, the parameter λ_k is introduced in Eq. (5) [34,35]. In particular, when $\lambda_k = 0$ and $\boldsymbol{J}_k$ is nonsingular, the LM method will be changed to Gauss–Newton (Eqs. (7) and (8)):

$$\boldsymbol{\phi}_k^{GN} = -(\boldsymbol{J}_k^T\boldsymbol{J}_k)^{-1}\boldsymbol{J}_k^T\boldsymbol{F}_k \tag{7}$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{\phi}_k^{GN} \tag{8}$$

Ref. [36] has suggested a corrected LM technique for the singular value system of nonlinear equations and [37] recommended a modified LM technique. At each iteration, the modified LM technique firstly obtains $\boldsymbol{\phi}_k^{LM}$ by solving the following algebraic linear Eqs. (9)–(11).

$$\lambda_k = \mu_k\|\boldsymbol{F}_k\|^{\delta} \tag{9}$$

where $\delta \in (0,2]$ and $\mu_k > 0$

$$\vartheta_k^{LM} = -(\boldsymbol{J}_k^T\boldsymbol{J}_k + \lambda_k\boldsymbol{I})^{-1}(\boldsymbol{J}_k^T\boldsymbol{F}_k + \lambda_k\boldsymbol{\phi}_k^{LM}) \tag{10}$$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \vartheta_k^{LM} \tag{11}$$

Ref. [38] has proposed a LM technique with a non-monotone second-order line search, which can be written as follows: