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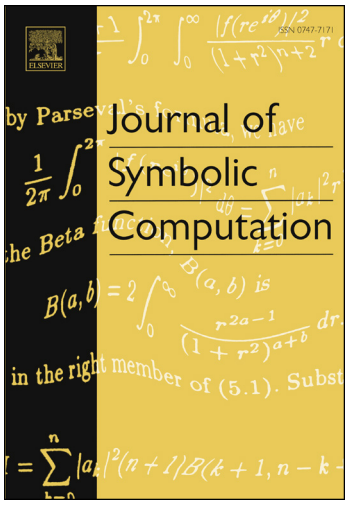

Algorithmic operator algebras via normal forms in tensor rings

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Algorithmic operator algebras via normal forms in tensor rings[1]

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Abstract

We propose a general algorithmic approach to noncommutative operator algebras generated by additive operators using quotients of tensor rings that are defined by tensor reduction systems. Skew polynomials are a well-established tool covering many cases arising in applications. However, integro-differential operators over an arbitrary integro-differential algebra do not fit this structure, for example. Instead of using parametrized Gröbner bases in free algebras, as has been used so far in the literature, we use Bergman's basis-free analog in tensor rings. Since reduction rules are given by module homomorphisms, the tensor setting often allows for a finite reduction system. A confluent tensor reduction system enables effective computations based on normal forms. Using tensor rings, we can also model integro-differential operators with matrix coefficients, where constants are not commutative.

To have smaller reduction systems, we develop a generalization of Bergman's setting. It allows overlapping domains of reduction homomorphisms, which also make the algorithmic verification of the confluence criterion more efficient. Moreover, we discuss a heuristic approach to complete a given reduction system to a confluent one in analogy to Buchberger's algorithm and Knuth-Bendix completion. Integro-differential operators are used to illustrate the tensor setting, verification of confluence, and completion of tensor reduction systems. We also introduce a confluent reduction system and normal forms for integro-differential operators with linear substitutions, which have applications in delay differential equations. Verification of the confluence criterion and completion based on S-polynomial computations is supported by the Mathematica package `TenReS`.

Keywords: operator algebra; tensor ring; integro-differential operators; linear substitutions; noncommutative Gröbner basis; reduction systems; completion; confluence

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