

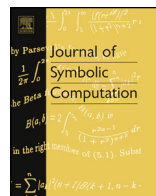


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Dancing samba with Ramanujan partition congruences



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ABSTRACT

The article presents an algorithm to compute a $C[t]$ -module basis G for a given subalgebra A over a polynomial ring $R = C[x]$ with a Euclidean domain C as the domain of coefficients and t a given element of A . The reduction modulo G allows a subalgebra membership test. The algorithm also works for more general rings R , in particular for a ring $R \subset C((q))$ with the property that $f \in R$ is zero if and only if the order of f is positive. As an application, we algorithmically derive an explicit identity (in terms of quotients of Dedekind η -functions and Klein's j -invariant) that shows that $p(11n + 6)$ is divisible by 11 for every natural number n where $p(n)$ denotes the number of partitions of n .

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1. Introduction

Ramanujan (1921) discovered that

$$p(5n + 4) \equiv 0 \pmod{5} \quad (1)$$

$$p(7n + 5) \equiv 0 \pmod{7} \quad (2)$$

$$p(11n + 6) \equiv 0 \pmod{11} \quad (3)$$

for all natural numbers $n \in \mathbb{N}$ where $p(n)$ denotes the number of partitions of n . In Ramanujan (1919) he lists the following identities from which (1) and (2) can be concluded.

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$$\sum_{n=0}^{\infty} p(5n + 4)q^n = 5 \prod_{k=1}^{\infty} \frac{(1 - q^{5k})^5}{(1 - q^k)^6} \tag{4}$$

$$\sum_{n=0}^{\infty} p(7n + 5)q^n = 7 \prod_{k=1}^{\infty} \frac{(1 - q^{7k})^3}{(1 - q^k)^4} + 49q \prod_{k=1}^{\infty} \frac{(1 - q^{7k})^7}{(1 - q^k)^8} \tag{5}$$

A similar “simple” identity for (3) is not known, although [Lehner \(1943\)](#) gave an identity in terms of ad hoc constructed series A and C .

$$q \prod_{k=1}^{\infty} (1 - q^{11k}) \sum_{n=0}^{\infty} p(11n + 6)q^n = 11(11AC^2 - 11^2C + 2AC - 32C - 2)$$

[Radu \(2015\)](#) developed an algorithmic machinery based on modular functions. He first computed generators M_1, \dots, M_7 of the monoid of all quotients of Dedekind η -functions of level 22 that only have poles at infinity. For more details see [Radu \(2015\)](#). In terms of q -series, these generators are as follows.

$$M_1 = q^{-5} \prod_{k=1}^{\infty} \frac{(1 - q^k)^7 (1 - q^{11k})^3}{(1 - q^{2k})^3 (1 - q^{22k})^7}$$

$$M_2 = q^{-5} \prod_{k=1}^{\infty} \frac{(1 - q^{2k})^8 (1 - q^{11k})^4}{(1 - q^k)^4 (1 - q^{22k})^8}$$

$$M_3 = q^{-6} \prod_{k=1}^{\infty} \frac{(1 - q^{2k})^6 (1 - q^{11k})^6}{(1 - q^k)^2 (1 - q^{22k})^{10}}$$

$$M_4 = q^{-5} \prod_{k=1}^{\infty} \frac{(1 - q^{2k})(1 - q^{11k})^{11}}{(1 - q^k)(1 - q^{22k})^{11}}$$

$$M_5 = q^{-7} \prod_{k=1}^{\infty} \frac{(1 - q^{2k})^4 (1 - q^{11k})^8}{(1 - q^{22k})^{12}}$$

$$M_6 = q^{-8} \prod_{k=1}^{\infty} \frac{(1 - q^k)^2 (1 - q^{2k})^2 (1 - q^{11k})^{10}}{(1 - q^{22k})^{14}}$$

$$M_7 = q^{-9} \prod_{k=1}^{\infty} \frac{(1 - q^k)^4 (1 - q^{11k})^{12}}{(1 - q^{22k})^{16}}$$

Note that each of these series lives in $\mathbb{Z}((q))$. By his algorithms AB and MW, Radu then computes a relation

$$F = 11(98t^4 + 1263t^3 + 2877t^2 + 1019t - 1997) + 11z_1(17t^3 + 490t^2 + 54t - 871) + 11z_2(t^3 + 251t^2 + 488t - 614) \tag{6}$$

where F is defined as on top of p. 30 of [Radu \(2015\)](#), i.e.,

$$F = q^{-14} \prod_{k=1}^{\infty} \frac{(1 - q^k)^{10} (1 - q^{2k})^2 (1 - q^{11k})^{11}}{(1 - q^{22k})^{22}} \sum_{n=0}^{\infty} p(11n + 6)q^n \tag{7}$$

and t, z_1, z_2 are given by

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