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Nonnegative low-rank representation based manifold embedding for semi-supervised learning

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ABSTRACT

The low-rank representation (LRR) can get essential row-representation of data and it is robust to illumination variation, occlusions and other types of noise. This paper presents a novel manifold embedding classification algorithm based on nonnegative low-rank representation for semi-supervised learning (MEC-NNLRR). In the proposed algorithm, label fitness, manifold smoothness and low-rank representation are integrated, and the label information from the labeled data and the manifold structure from all data are fully and effectively utilized. Based on LRR and manifold learning, the proposed MEC-NNLRR can capture the global and local structure information of the observed data. The obtained nonnegative low-rank representation coefficients can be used as a graph similarity matrix. Considering the physical interpretation of the graph matrix, we impose the non-negativity constraint on the coefficients. In addition, no matter whether the training samples or test samples are corrupted, the proposed MEC-NNLRR is little affected by noise. Extensive experiments on public image databases demonstrate that the proposed MEC-NNLRR is an excellent algorithm and achieves satisfactory results.

1. Introduction

Biometrics is still one of the research hotspots in computer vision and artificial intelligence. Because face recognition is simple and non-contact, it has been widely studied in the past decades. However, it is still very difficult to understand because of its high dimensionality. As we can see, the time and memory consumption is usually unacceptable to deal with the high-dimension data, which is hard to be processed by some existing algorithms. Dimension reduction can obtain efficient low dimensional representation of high-dimension data, which is helpful for calculation, classification, storage and visualization. Therefore, a lot of algorithms for dimension reduction have been proposed [1–5]. The most classical linear dimensional reduction algorithms are PCA and LDA. PCA is an unsupervised dimension reduction method, which does not use the class label information of the observed data. LDA is a supervised method by utilizing the class label information in the feature extraction process, which is useful for classification recognition task [6]. If enough labeled data are available, the recognition performance of the supervised methods is usually better than that of the unsupervised methods.

The image data usually exists in a nonlinear low-dimensional sub-manifold space which is hidden in the original high-dimensional image space. However, the intrinsic nonlinear structure of the observed data is usually difficultly to be properly discovered by the linear dimension reduction methods. In order to reveal the essential nonlinear manifold structure of image data, many nonlinear manifold learning algorithms have been presented [7–9]. Locally linear embedding (LLE) [7], ISOMAP [8] and Laplacian eigenmaps [9] are three most representative manifold learning algorithms. These manifold methods can reveal the essential structure of the data effectively and achieve satisfactory performance. However, these methods usually suffer from the so-called out-of-sample problem. That is to say, no projection matrix can be available in these methods. When we have a new image data, we have to retrain all the image samples. This is very time consuming. Thus the methods are not suitable for real time recognition and classification. In order to solve this problem, many improved manifold learning algorithms have been proposed [10–16]. A patch alignment based manifold learning framework was proposed in [10], which included two stages: part optimization and whole alignment. Chen et al. [11] presented a supervised orthogonal discriminant subspace projection (SODSP) algorithm. In SODSP, a new weight matrix is constructed, and the neighborhood

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structure information is kept. At the same time, an orthogonal constraint is imposed into a graph based maximum margin analysis algorithm. In order to deal with the occlusions and illumination variations, a manifold regularized local sparse representation (MRLSR) method was proposed [12]. It is the key idea for MRLSR to keep group sparsity. He et al. [13] presented a locality preserving projection (LPP) algorithm. In LPP, the obtained projection matrix can describe well the image manifold structure. Cai et al. [14] presented a locality sensitive discriminant analysis (LSDA) algorithm. Yan et al. [15] gave a unified graph embedding framework, in which a novel marginal fisher analysis (MFA) algorithm was proposed. Both the intrinsic graph and the penalty graph were built respectively in MFA. A novel face image recognition method [16] was presented in which the coupled mappings are firstly established, and then they are used to respectively project the low-resolution images and high-resolution images to a unified latent subspace.

Recently, the clean essential low-rank data matrix can be obtained from the corrupted observed data matrix by low-rank recovery technique, which is attracting more and more researchers' attention. We usually encounter two problems in image clustering: how the samples from the different subspaces can be correctly clustered into their respective subspace and how it gets rid of possible outliers. In order to solve the two problems, Liu et al. [17] proposed a low-rank representation (LRR) method. In LRR, the lowest rank representation of all observed data can be obtained by solving the nuclear norm optimization problem. Zhang et al. [18] presented a regularized low-rank representation (rLRR) framework, which can effectively preserve the similarities of principal and salient features. Based on the rLRR framework, low-rank similarity preserving projections (LSPP) are further presented for feature extraction. By combining low-rank representation with graphs and kernel trick, a novel semi-supervised kernel low-rank representation graph (SKLRG) is proposed to achieve robust classification for wide range of datasets [19]. Previous study [20] showed that the nonnegative constraint in LRR not only allowed people to attain interpretable representation coefficients but also can led to perfect result. In order to discover the essential structures of the data, a nonnegative low-rank and sparse (NNLRS) graph algorithm was proposed [21], in which both the global structure information and the local structure information can be captured. Lu et al. [22] proposed a graph-regularized low-rank representation destriping method, which can effectively remove the effects of the striping noise. Xu et al. [23] presented a discriminative transfer subspace learning method based on low-rank and sparse representation, in which the problem about unsupervised domain transfer learning is well addressed. In order to fully use the geometrical structure of data, Peng et al. [24] presented a manifold low-rank representation (MLRR) algorithm.

Because the class label information is utilized in the supervised learning methods, they usually outperform the unsupervised learning methods. We have only a small amount of labeled data in the practical application. This is because it takes a large amount of time to collect and organize the labeled data. However, there are still lots of unlabeled data in real life, and they can be easily obtained. In order to take full use of the limited labeled data and the abundant unlabeled data for better classification recognition, many semi-supervised algorithms were proposed [25–28]. A unified framework was proposed for semi-supervised and unsupervised learning, in which the new samples can be mapped and effectively classified [25]. Gao et al. [26] proposed a stable semi-supervised discriminant learning method (SSDL), in which the essential structure can be found. In semi-supervised learning, it is usually seldom studied for graph construction. In order to solve this problem, Yu et al. [27] proposed a novel semi-supervised classification method based on random subspace dimensionality reduction (SSC-RSDR). Zhao et al. [28] proposed a semi-supervised learning algorithm, in which both the global and local discriminative information are well preserved.

Inspired by manifold learning and low-rank representation, we present a novel manifold embedding classification algorithm based on nonnegative low-rank representation (MEC-NNLRR) for semi-supervised learning. In the proposed MEC-NNLRR, both the global structure information and local structure information of observed data are fully taken into account in graph construction. The global structure can be well emphasized by the low-rank constraint, and the local structure is preserved by the manifold, which is composed of the labeled data and a great deal of unlabeled data.

The main contributions of the proposed algorithm are listed as follows.

Firstly, label fitness, manifold smoothness and nonnegative constraint are integrated into a framework. Secondly, the label information from the labeled data and the manifold structure information from all data are fully and effectively utilized. Thirdly, both the graph construction and label prediction are carried out at the same time. Lastly, the proposed MEC-NNLRR is robust to noise.

This paper is organized as follows. In Section 2, the related works are briefly reviewed. In Section 3, the proposed method is described in detail. Section 4 discusses the experimental results on different face databases. The final section gives our conclusions.

2. Related works

In this section, we briefly review the references related to our proposed algorithm, which mainly include Gaussian fields and harmonic functions (GFHF) [29,30] and low-rank representation (LRR) [17]. GFHF is an effective method to deal with semi-supervised learning, and it is easy to be integrated with other methods, such as graph learning [31–33], to obtain very promising results [34]. For semi-supervised learning, GFHF can propagate labels from labeled samples to unlabeled ones in a mathematically tractable way.

2.1. Gaussian fields and harmonic functions (GFHF)

Suppose an observed data set $A = [a_1, a_2, \cdots, a_m, a_{m+1}, \cdots, a_n] \in R^{d\times n}$ from c classes. There are m labeled samples and $n-m$ unlabeled samples, where $a_i\ (i = 1, \cdots, m)$ is the labeled data, $a_i\ (i = m+1, \cdots, n)$ is the unlabeled data, and d represents dimension. We define a label matrix $Y \in R^{n\times c}$ as follows.

$$Y_{ij} = \begin{cases} 1 & y_i = j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $y_i \in \{1, 2, \cdots, c\}, i = 1, 2, \cdots, m$.

$G = \{A, W\}$ is an undirected weighted graph, where $W \in R^{n\times n}$ is a similarity matrix. The graph Laplacian matrix L is defined as

$$L = D - W \tag{2}$$

where D is a diagonal matrix, and its diagonal elements can be obtained as follows.

$$D_{ii} = \sum_{j=1}^{n} W_{ij} \quad (i = 1, 2, \cdots, n) \tag{3}$$

The prediction label matrix is denoted by $F \in R^{n\times c}$. The first m vectors from F are required to be close to the class label of labeled data. At the same time, the label matrix F should be as smooth as possible on the whole graph, which is consisted of labeled samples and unlabeled samples. The objective function of Gaussian fields and harmonic functions (GFHF) is given as follows.

$$g(F) = \frac{1}{2} \sum_{\substack{i=1 \\ j=1}}^{n} \left\| F_{i\cdot} - F_{j\cdot} \right\|^2 W_{ij} + \lambda_\infty \sum_{i=1}^{m} \| F_{i\cdot} - Y_{i\cdot} \|^2 \tag{4}$$