Knowledge-Based Systems 122 (2017) 51–63

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Matrix-based approaches for dynamic updating approximations in multigranulation rough sets

Chengxiang Hu*, Shixi Liu, Guoxiu Liu

School of Computer and Information Engineering, Chuzhou University, Chuzhou 239000, China

ARTICLE INFO

Article history:
Received 6 October 2016
Revised 21 December 2016
Accepted 21 January 2017
Available online 23 January 2017

Keywords:
Knowledge discovery
Updating approximations
Multigranulation
Matrix

ABSTRACT

Multigranulation rough set, which is constructed by a family of equivalence relations has attracted much attention, because it offers a theoretical framework for the problem solving in the view of multigranulation. However, the granular structure in the information systems often dynamically evolves over time. How to dynamically obtain the potential useful knowledge for decision making is of great significance in the context of multigranulation. Motivated by this requirement, in this paper, we present the definitions of equivalence relation matrix, diagonal matrix and cut matrix for a single granular structure in multigranulation rough set, and propose a matrix representation of multigranulation approximations in optimistic and pessimistic multigranulation rough set. Then, corresponding matrix-based dynamic approaches for updating approximations are proposed in multigranulation rough set when a single granular structure evolves over time. The experimental evaluations show the effectiveness of the proposed matrix-based dynamic updating algorithms compared with the matrix-based static algorithm.

1. Introduction

Rough set theory, which was originally proposed by Pawlak in 1980s, is a powerful mathematical tool for data analysis and knowledge processing [27,28]. It has been widely used in image precessing, machine learning, pattern recognition, data mining and other relevant areas. With respect to various requirements, several extensions of classical rough set model have been proposed [10,37,44–47,54], such as variable precision rough set, rough set model in incomplete information systems, probabilistic rough sets, composite rough sets and multigranulation rough set.

As is well-known that, classical rough set theory is constructed by the single equivalence relation, and it is too restrictive in many real application areas. In order to extend the application areas of rough set theory, Qian et al. proposed multigranulation rough set which includes optimistic multigranulation rough set and pessimistic multigranulation rough set [31]. The universe is partitioned as a family of equivalence relations in multigranulation rough set. Since then, multigranulation rough set has become an important research topic, and many extensions of multigranulation rough set model have been proposed. For instance, Lin et al. discussed the relationship between multigranulation rough set theory and the evidence theory and proposed a fusion approach for dealing with uncertain data from multi-source systems [17]. Qian et al. extended multigranulation rough set from the equivalence relation to the tolerance relation [29]. Furthermore, Qian et al. developed multigranulation decision-theoretic rough set model using probabilistic theory and multigranulation rough set [32]. Lin et al. presented covering-based multigranulation rough set and developed some properties and uncertainty measures of the covering-based multigranulation rough set [16]. To deal with the neighborhood information systems with heterogeneous attributes, Lin et al. developed neighborhood-based multigranulation rough set [18]. Yang et al. presented multigranulation rough sets in incomplete information systems based on tolerance and limited tolerance relations [43]. She et al. investigated the topological structure of multigranulation rough set [34]. Feng et al.proposed variable precision multigranulation decision-theoretic fuzzy rough set [6].

With the development of information technology, information systems dynamically vary in many real applications. How to effectively acquire useful information from dynamic information systems has attracted much attention in recent years. In order to deal with dynamic information systems, many incremental approaches for knowledge acquisition have been proposed by researchers. Based on the theories of granular computing and rough sets, Li et al. presented incremental knowledge discovery methods to handle big data with rich information [11,12]. From the viewpoint of rough set theory, incremental learning approaches mainly

* Corresponding author.
E-mail addresses: chengxiang0550@163.com (C. Hu), liusxchuz@163.com (S. Liu), liuxiu.03@163.com (G. Liu).

http://dx.doi.org/10.1016/j.knosys.2017.01.030
0950-7051/

focused on the following three aspects in dynamic information systems.

(1) Dynamic updating approaches under the variation of objects. Chen et al. presented a rough set approach for updating approximations in variable precision rough set when objects alter with time [5]. Hu et al. proposed knowledge maintenance approach for updating approximations in rough set model over dual-universes [7]. In addition, they developed incremental method for updating fuzzy probabilistic rough approximations with incremental objects [8]. Luo et a. presented dynamic approaches for updating approximations in probabilistic rough set model with increasing of objects [23]. Luo et al. proposed an approach for updating approximation while objects are added into or deleted from set-valued ordered information systems [24]. Based on Bayesian decision procedure under dynamic granulation, Sang et al. proposed a method for updating parameters of each granular [33]. Shu et al. developed an incremental approach for attribute reduction in dynamic incomplete information systems [36]. Yu et al. developed an approach for updating approximations in interval-valued decision information systems in dynamic environment while adding or removing objects [48].
(2) Dynamic updating approaches under the variation of attributes. Chan developed an algorithm for acquiring knowledge from large databases in order to deal with inconsistent data [1]. Li et al. presented incremental approaches for updating approximations of a concept when attributes are added into or deleted from incomplete information systems under characteristic relations [13]. Li et al. proposed incremental algorithms for updating approximations in dominance-based rough set when attributes are added into or deleted from information systems [15]. Liu et al. presented a block-based boundary sets for updating approximations in probabilistic rough sets under the variation of attributes [20]. Shu et al. presented a positive region based approach for dynamic attribute reduction when attributes vary [35]. Qian et al. proposed a theoretic framework of positive approximation for attribute reduction, which can effectively obtain reduction [30]. Zhang et al. proposed incremental approaches for updating approximations under dynamic variation of attributes in interval-valued information systems [52]. Yang et al. proposed the methods for updating approximations and attribute reduction in multigranulation rough set under the increasing of granular structures [42]. Ju et al. presented approaches for updating approximations and attribute reduction in multigranulation fuzzy rough sets when adding granular structures [9].
(3) Dynamic updating approaches under the variation of attribute values. Chen et al. developed dynamic approaches for updating approximations while refining or coarsening attribute values, which can effectively reduce the computational time [3]. Chen et al. proposed incremental approaches for updating approximations of upward and downward union in dynamic incomplete information systems while attribute values are coarsening or refining [4]. Luo et al. investigated incremental approaches for updating rough approximations with the single value varies in set-valued decision systems[25]. Wang et al. proposed a dynamic attribute reduction approach based on information entropy under the variation of attribute values [39]. Li et al. proposed an approach for dynamic updating approximations in dominance-based rough sets when some attribute values vary [14]. Zeng et al. developed incremental updating approximations approaches in fuzzy rough sets under the variation of attribute values [53].

Generally speaking, matrix-based technique is a simple and effective method for dealing with complicated data, which can be effectively used for dynamic knowledge acquisition. Zhang et al. proposed a parallel matrix-based approach for computing approximations in incomplete information systems [51]. Wang et al. presented a boolean matrix approach for description of covering approximation operators, and proposed a novel viewpoint to investigate covering-based rough set [41]. Tan et al. proposed two kinds of descriptions in covering decision systems based on matrix, which are used into reduction and computing approximations [38]. Liu explored a matrix-based method for lower and upper approximation operation of rough set [21]. Chen et al. proposed an approach for updating knowledge when objects and attributes simultaneously vary in decision-theoretic rough sets [2]. Liu et al. proposed a matrix-based method for illustrating the incremental processing in knowledge discovery [19]. Zhang et al. proposed composite information systems which include attributes of multiple different types, and presented matrix-based incremental methods for updating approximations with the variation of objects [49]. Zhang et al. proposed a matrix-based incremental algorithm for updating approximations in set-valued information systems with the variation of attributes [50]. Luo et al. presented matrix-based dynamic approaches for updating approximations while the attributes are added into and deleted from set-valued ordered information systems [22]. Luo et al. proposed matrix-based incremental approaches for updating approximations under the addition and the deletion of objects in decision-theoretic rough sets [26]. Wang et al. developed an incremental mechanism for updating approximations in dominance-based rough sets under simultaneous increase of objects and attributes [40].

However, to the authors' best knowledge, matrix-based approaches for dynamic updating approximations in multigranulation rough set have not been discussed so far. The main objective of this paper is to present matrix-based approaches for updating lower and upper approximations in the context of multigranulation rough set when a single granular structure is added into or deleted from information systems. Firstly, we introduce the definitions of equivalence relation matrix, diagonal matrix and cut matrix in terms of a single granular structure. Then, we present matrix-based dynamic algorithms for updating approximations in dynamic information systems when a single granular structure is added into or deleted from information systems. Finally, experimental results are performed to show the efficiency of the proposed matrix-based dynamic algorithms.

The main contributions of this paper can be summarized as follows: (1) A matrix-based approach is proposed for calculating lower and upper approximations in the context of multigranulation rough set. (2) The matrix-based dynamic algorithms are developed for updating multigranulation approximations when a single granular structure is added into or deleted from information systems. (3) The computational efficiency of the proposed dynamic algorithms outperforms the static algorithm.

The rest of this paper is organized as follows. In Section 2, we review several concepts in multigranulation rough set, and propose the matrix representation of lower and upper approximations in multigranulation rough set. In Section 3, we present a matrix-based dynamic algorithm for updating approximations when a single granular structure is added into information systems. In Section 4, we propose a matrix-based dynamic algorithm for updating approximations in optimistic and pessimistic multigranulation rough set when a single granular structure is deleted from information systems. In Section 5, experimental evaluations are conducted to verify the effectiveness of the proposed algorithms in comparison with the static algorithm. The paper ends with conclusion and further research directions in Section 6.