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Relaxed observer design of discrete-time nonlinear systems via a novel ranking-based switching mechanism

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ABSTRACT

In this paper, the problem of relaxed observer design of discrete-time nonlinear systems is studied by developing a novel ranking-based switching mechanism. To do this, the useful ranking information of the normalized fuzzy weighting functions is utilized in order to give a denser subdivision of the normalized fuzzy weighting function space and therefore essentially yields the proposed ranking-based switching mechanism. Based on the obtained switching mechanism, a family of switching observers can be developed for the purpose of guaranteeing the estimation error system to be asymptotically stable with less conservatism than the existing results available in the references. Finally, two numerical examples are presented to illustrate the advantages of the proposed method.

1. Introduction

Takagi–Sugeno (T–S) fuzzy models based on the sector nonlinearity approach [1] has been widely applied since they can exactly capture the bounded nonlinearities within a compact set of the systems state space. As a result of their usability, the problem of control synthesis based on T–S fuzzy models has been intensively studied and a great deal of important results have been proposed, such as adaptive fuzzy control designs [2–5], fuzzy state/output feedback control designs [6–10] and fuzzy-model-based fault detections [11–13]. However, in view of the fact that not all of the system state variables are available in actual control systems, the aforementioned results usually fail to work only if the problem of state estimations for unavailable system state variables could be well solved [14,15]. As far as T–S fuzzy model-based state estimations are concerned, some featured results have been proposed, e.g., filtering of continuous-time T–S fuzzy systems [16–18], non-fragile H_∞ filtering [19–21], filtering of discrete-time T–S fuzzy systems [22,23], filtering of T–S fuzzy systems with time delays [24–26], and fuzzy model-based model reduction [27–29].

Which deserving additional attention is, the aforementioned results are based on the parallel distributed compensation theory, and then the obtained fuzzy observers/filters are only dependent on the current-time normalized fuzzy weighting functions. Consequently, the obtained design conditions are rather conservative [30]. Recently, inspired by the so-called multi-instant matrix theory, a novel fuzzy observer that is parameter-dependent on multi-steps normalized fuzzy weighting functions has been provided for the purpose of achieving less conservative results in [31]. More recently, the result of [31] has been further relaxed in [32] by introducing the so-called 'Slack Variable Approach'. Now, we may naturally raise a question: whether the conservatism will further reduced if more algebraic properties of the normalized fuzzy weighting functions are further fused into the process of future observer designs? That is to say, the research of exploring more efficient design conditions of fuzzy state estimations is still very challenging, which motivates us to present this study.

In order to reducing the conservatism of the existing results, relaxed observer design conditions of discrete-time nonlinear systems are proposed by employing an efficient ranking-based switching mechanism. Using the ranking information of the normalized fuzzy weighting functions has a beneficial effect on giving a denser subdivision of the normalized fuzzy weighting function space and therefore essentially yields the proposed ranking-based switching mechanism. With the help of the proposed ranking-based switching mechanism, a kind of

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switching observer is developed and thus the estimation error system is ensured to be asymptotically stable with less conservatism than previous ones available in the literature.

The rest of the study is arranged as follows: Background and key preliminaries are provided in Section 2. In Section 3, the main result is proposed. In Section 4, two illustrative numerical examples are presented to show the advantages of the proposed method. Finally, some conclusions are provided and thus completes the paper in Section 5.

Notations. In this paper, $\mathbb{R}$ stands for the set of real numbers, $\mathbb{Z}_+$ represents the set of positive integers, $\mathbb{N}$ denotes the set of natural numbers, i.e., $\{0, 1, 2, \ldots\}$. Moreover, $p\,!$ represents the factorial, i.e., $p\,! = p(p-1)(p-2)\cdots(2)(1)$ for $p \in \mathbb{N}$, and in particular one has $0\,! = 1$. Also, we define the left-hand side of a relation as Left$(\cdot)$, and He(E) stands for $E + E^T$.

2. Background and key preliminaries

A family of discrete-time nonlinear systems can be represented by the following T–S fuzzy model [1]:

$$\begin{cases} x(t+1) = \displaystyle\sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \\ y(t) = \displaystyle\sum_{i=1}^{r} h_i(z(t))\{C_i x(t)\} \end{cases} \tag{1}$$

In detail, $x(t) \in \mathbb{R}^{n_1}$ stands for the system state vector, $u(t) \in \mathbb{R}^{n_2}$ stands for the control input vector, $y(t) \in \mathbb{R}^{n_3}$ stands for the system output vector, $z(t)$ denotes the available premise variable, and $h_i(z(t))$ stands for the ith current–time normalized fuzzy weighting function, $A_i \in \mathbb{R}^{n_1 \times n_1}$, $B_i \in \mathbb{R}^{n_1 \times n_2}$, $C_i \in \mathbb{R}^{n_3 \times n_1}$ are a bank of prior known system parameter matrices.

More importantly, it is worth noticing that not all of the system state variables are available in actual control systems. As a result if this reason, the problem of state estimations for unavailable system state variables have attracted a lot of attention, e.g. those results given in [30–32]. In this study, our main objective is to propose a bank of more relaxed observer design conditions such that the unavailable system state variables can be estimated by employing the available $y(t)$.

Next, let's give a bank of important notations about homogeneous polynomials which are fairly standard as those applied in [31,32].

The set Δ_r is defined as $\Delta_r = \{\alpha \in \mathbb{R}^r; \sum_{i=1}^{r} \alpha_i = 1; \alpha \geq 0\}$. $\alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_r^{k_r}$, $\alpha \in \Delta_r$, $k_i \in \mathbb{Z}_+$, $i = 1, 2, \ldots, r$ are defined as the monomials, $k = k_1 k_2 \cdots k_r$, and $P_k \in \mathbb{R}^{n \times n}$, $\forall k \in \mathcal{K}(g)$ are matrix-valued coefficients. Here, $\mathcal{K}(g)$ represents the set of r-tuples obtained as all possible combinations of nonnegative integers k_i, $i = 1, 2, \ldots, r$, satisfying $\sum_{i=1}^{r} k_i = g$. By definitions, $\pi(k) = (k_1\,!)(k_2\,!)\cdots(k_r\,!)$, $\chi_i = 0\cdots0\underbrace{1}_{i\text{–th}}0\cdots0$.

Particularly, we write $k \geq k'$ if $k_i \geq k'_i$, $(i = 1, \ldots, r)$. The usual operations of summation, $k + k'$, and subtraction, $k - k'$ (whenever $k \geq k'$), are defined componentwise. Furthermore, some shortenings are utilized in this study as follows:

$$\begin{cases} h(z(t-j)) = \{h_1(z(t-j)), \cdots, h_r(z(t-j))\}^T, \quad j \in \{-1, 0, 1, 2, \ldots\}, \\ h^k(z(t-j)) = h_1^{k_1}(z(t-j)) \times h_2^{k_2}(z(t-j)) \times \cdots \times h_r^{k_r}(z(t-j)). \end{cases} \tag{2}$$

Finally, an important lemma is also mentioned here.

Lemma 1. [33]. *For two symmetric matrices $P > 0$ and $P_+ > 0$, the matrix inequality $A^{\mathrm{T}} P_+ A - P < 0$ holds, if there is another matrix G such that*

$$\begin{bmatrix} P & (*) \\ GA & G + G^{\mathrm{T}} - P_+ \end{bmatrix} > 0.$$

3. Main results

3.1. A novel ranking-based switching mechanism

In order to state the idea of ranking-based switching mechanism clearly, a simple example is given to make the issue easier to understand.

If $r = 3$, then the normalized fuzzy weighting function space can be divided into six sub-spaces, i.e., $h_1(z(t)) \geq h_2(z(t)) \geq h_3(z(t)) \geq 0$, $h_1(z(t)) \geq h_3(z(t))) \geq h_2(z(t)) \geq 0$, $h_2(z(t)) \geq h_1(z(t)) \geq h_3(z(t)) \geq 0$, $h_2(z(t)) \geq h_3(z(t)) \geq h_1(z(t)) \geq 0$, $h_3(z(t)) \geq h_1(z(t)) \geq h_2(z(t)) \geq 0$ and $h_3(z(t)) \geq h_2(z(t)) \geq h_1(z(t)) \geq 0$.

Obviously, different normalized fuzzy weighting function plays different role in any chosen sub-space. For instance, $h_1(z(t))$ plays the most important role in the sub-space generated by $h_1(z(t)) \geq h_2(z(t)) \geq h_3(z(t)) \geq 0$ while $h_3(z(t))$ plays the least important role in it. Now, with an idea in mind, if the ranking information could be well utilized for observer design, then the conservatism of the existing results will be reduced. Therefore, a ranking-based switching mechanism is proposed as the following two steps:

(1) sorting a set of $\{h_i(z(t)), i \in \{1, 2, \ldots, r\}\}$ in descending order and recording their indexes (i.e., $i \in \{1, 2, \ldots, r\}$) sequentially by $\{\text{num}(j), j \in \{1, 2, \ldots, r\}\}$. In particular, in case of $h_{j_1}(z(t)) = h_{j_2}(z(t))$, one can sort them in descending order according to the values of j_1 and j_2.
(2) choosing the operating mode according to the obtained $\{\text{num}(j), j \in \{1, 2, \ldots, r\}\}$. Indeed, there are $(r\,!)$ kinds of possible combinations of $\{\text{num}(j), j \in \{1, 2, \ldots, r\}\}$, i.e., one can select a proper operating mode among these $(r\,!)$ kinds of operating modes while the ranking information is well utilized.