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On global exponential stability of positive neural networks with time-varying delay

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Abstract

This paper presents a new result on the existence, uniqueness and global exponential stability of a positive equilibrium of positive neural networks in the presence of bounded time-varying delay. Based on some novel comparison techniques, a testable condition is derived to ensure that all the state trajectories of the system converge exponentially to a unique positive equilibrium. The effectiveness of the obtained results is illustrated by a numerical example.

Keywords: Positive neural networks, positive equilibrium, exponential stability, time-varying delay.

1. Introduction

Asymptotic behavior of neural networks (NNs) is an important research topic. Over the past few decades, the problem of stability analysis of NNs has received considerable attention due to its widespread applications in signal processing, pattern recognition, ecosystem evaluation and parallel computation [1–3]. In such applications, it is important to ensure the network has a unique equilibrium which is globally stable [4]. Besides that time delays are omnipresent in applied network models which usually lead to poor performance or complex behavior of the system [5]. Therefore, during the last decade, a great deal of effort from researchers has been devoted to the problem of stability analysis and control of delayed neural networks (DNNs) [6–10].

On the other hand, in modeling of many applied models in economics, ecology and biology or communication systems, the relevant state variables such as messenger ribonucleic acids (mRNAs) which cellular organisms use to convey genetic information, proteins and molecules, electric charge or light intensity levels are subject to positivity constraints according to the nature of the phenomenon itself [11, 12]. Such systems are commonly referred to as positive systems. Positive systems belong to a class of dynamical systems whose states are always nonnegative whenever the inputs and initial conditions are nonnegative [11]. As an essential issue in applications of positive systems, the problem of stability analysis and control of positive systems and, in particular, positive systems with delays, has received considerable attention from researchers in the past few decades [13–17]. While this problem has been thoroughly investigated for many classes of DNNs, the theory for positive delayed neural networks (PDNNs) is considerably less well-developed. On one hand, when a neural network model is designed for practical positive systems, for example, in identification [18], control [19] or competitive-cooperation dynamical systems for decision rules, pattern formation, and parallel memory storage [20], it is inherent that the states of the designed networks are nonnegative. On the other hand, the nonlinearity of activation functions and the negativeness of self-feedback terms make the study of PDNNs more complicated. Thus, it is of interest to study the problem of stability analysis for PDNNs. To date, there are only a few results concerning stability of PDNNs. Particularly, in [20], the problem of asymptotic behavior analysis was studied for a class of positive Cohen–Grossberg neural networks with a common activation function and constant delay. Based on the linear matrix inequalities (LMIs) approach, a sufficient condition was derived for the existence, uniqueness and $\mathbb{R}^n_+$-asymptotic stability of a so-called nonnegative equilibrium of the system in the sense of nonlinear complement problem (NCP). Specifically, under the derived conditions, it was shown in [20] that all state trajectories of the system initiated from the first orthant $\mathbb{R}^n_+$ will converge to the unique solution of an associated NCP which, in general, is not an equilibrium of the system. The problem of positive almost periodic solutions was considered in [21] recurrent neural networks with both bounded time-varying delays and continuously distributed delays. By utilizing the Arzela-Ascoli convergence theorem, a set of conditions was derived to ensure the existence and uniqueness of a positive almost periodic solution which exponentially attracts all solutions of the system. However, the derived conditions in [21] do not ensure the positivity of the system for any nonnegative input vector and nonnegative initial function. In addition, the methods presented in [20, 21] cannot be extended to Hopfield positive NNs with time-varying delays which enspires us for the present study.

In this paper, we consider a class of positive NNs with bounded time-varying delay. Different from existing results, for example, in [20, 21], we derive conditions based on some novel comparison techniques to ensure simultaneously that the system is positive and, for each nonnegative input vector, there exists a unique nonnegative equilibrium point which is globally exponentially stable. The derived conditions are formulated in terms of linear programming which can be solved by various computational tools. Finally, the effectiveness of the obtained results is validated by a numerical example.

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