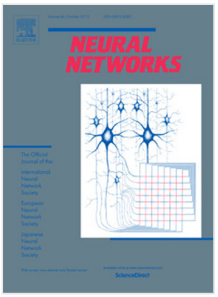
Accepted Manuscript



Neurons the decision makers, Part I: The firing function of a single neuron

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PII: S0893-6080(16)30033-8
DOI: http://dx.doi.org/10.1016/j.neunet.2016.04.003
Reference: NN 3613

To appear in: *Neural Networks*

Please cite this article as: Saaty, T. Neurons the decision makers, Part I: The firing function of a single neuron. *Neural Networks* (2016), http://dx.doi.org/10.1016/j.neunet.2016.04.003

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Neurons the Decision Makers:

Part I–The Firing Function of a Single Neuron

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Abstract

This paper is concerned with understanding synthesis of electric signals in the neural system based on making pairwise comparisons. Fundamentally, every person and every animal is born with the talent to compare stimuli from things that share properties in space or over time. Comparisons always need experience to distinguish among things. Pairwise comparisons are numerically reciprocal. If a value is assigned to the larger of two elements that have a given property when compared with the smaller one, then the smaller has the reciprocal of that value when compared with the larger. Because making comparisons requires the reciprocal property, we need mathematics that can cope with division. There are four division algebras that would allow us to use our reciprocals arising from comparisons: The real numbers, the complex numbers, the non-commutative quaternions and the non-associative octonions. Rather than inferring function as from electric flow in a network, in this paper we infer the flow from function. Neurons fire in response to stimuli and their firings vary relative to the intensities of the stimuli. We believe neurons use some kind of pairwise comparison mechanism to determine when to fire based on the stimuli they receive. The ideas we develop here about flows are used to deduce how a system based on this kind of firing determination works and can be described. Furthermore the firing of neurons requires continuous comparisons. To develop a formula describing the output of these pairwise comparisons requires solving Fredholm's equation of the second kind which is satisfied if and only if a simple functional equation has solutions. The Fourier transform of the real solution of this equation leads to inverse square laws like those that are common in physics. The Fourier transform applied to a complex valued solution leads to Dirac type of firings. Such firings are dense in the very general fields of functions known as Sobolev spaces and thus can be used to represent the very diverse phenomena in and around us. The non-commutative solution in quaternions can be interpreted as rotations in space. The also non-commutative and non-associative solution in octonions has yet to be adequately interpreted outside physics.