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Distributed optimization of first-order discrete-time multi-agent systems with event-triggered communication

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ABSTRACT

This paper focuses on the event-triggered distributed subgradient algorithms for solving a class of convex optimization problems based on first-order discrete-time multi-agent systems over undirected networks. The communication process of the whole network is controlled by a set of trigger conditions monitored by each agent. The trigger condition and event-triggered distributed subgradient optimization algorithm for each agent are completely decentralized and just rest with each agent's and its neighboring agents' individual states at the event-triggered sequence of themselves as well as each agent's local objective function. At each time instant, each agent updates its state by employing its own objective function and the states collected from itself and its neighboring agents at their separate event-triggered time instants. A sufficient condition for ensuring the consensus and reaching the optimization solution is established under the condition that the undirected network topology is connected and the design parameters are properly designed. Theoretical analysis shows that the event-triggered distributed subgradient algorithm is capable of steering the whole network of agents asymptotically converge to an optimal solution of the convex optimization problem. Simulation results validate effectiveness of the introduced algorithm and demonstrate feasibility of the theoretical analysis.

1. Introduction

In recent years, due to the great value in applications of multi-agent systems, more and more researchers in [1–12] have begun their research in this field and have obtained many remarkable achievements. Multi-agent system is not only a momentous class of system in the study of complex systems, but also an important branch of distributed artificial intelligence research. The coordination control of multi-agent systems is the frontier research direction in current field of control systems and has considerable application prospect in the field of intelligent robots [13], satellite formation flying [14], sensor networks [15–20], cloud computing [21–24], distributed measurement and monitoring [25–27], congestion control in communication network [28], and so on [29–33]. Different from the traditional control problems, a striking feature of the multi-agent system coordination control is that the control strategy is only based on the distributed control of local information between neighbors and achieve the desired overall goal through the information exchange. Multi-agent coordination control system consists of problems such as consensus [34], formation control [35] and distributed filtering [36]. Among these problems, the consensus problem for multi-agent systems is the core research branch since it is the basis of multi-agent collaboration and provides the theoretical support for other problems.

Event-triggered control strategies can effectively overcome the drawback of the traditional processing methods. This interesting area aroused considerable attention, see [37–48]. Tabuada [37] applied the event-triggered control method in control systems and provided the event-triggered control strategies, which not only guaranteed the asymptotic stability of the closed-loop system, but also further demonstrated that the event-triggered time sequence exclude the Zeno-like behavior. Building on the work of Tabuada [37], Dimarogonas et al. [39] primarily studied the consensus of first-order multi-agent systems in which the update of individual controller was based on the proportion of a certain measurement error with reference to the function with the Standard of the latest local states. Meanwhile, event-triggered control approach had a rapid development in consensus analysis for continuous-time multi-agent systems. Seyboth et al. [40] studied a variant of the event-triggered average consensus

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problem for single-integrator and double-integrators, in which a novel control strategy for multi-agent coordination was employed to simplify the performance and convergence analysis of the method. For multi-agent systems, the distributed rendezvous problem for single-integrator with combinational measurements was studied in Fan et al. [26]. However, it required that the Laplacian matrix of the associated communication topology ought to be symmetric. Furthermore, in [41] Li et al. considered the event-triggered distributed average-consensus of discrete-time first-order multi-agent systems with limited communication data rate. In the framework of network communication, although each agent has a real-valued state, they can only communicate finite-bit binary symbolic data sequence with its neighborhood agents at each time step on account of the digital communication channels with energy constraints. On the basis of the designed novel event-triggered dynamic encoder and decoder for each agent, a distributed control algorithm is proposed.

Recent work of Zhu et al. [42] had coordinately put their sights on the analysis of the event-triggered consensus problem which usually appeared in general linear time-invariant systems with fixed topology via applying an integral inequality technique. And a novel improved algorithm was introduced to determine the event time sequences, which can not only reduce communication between neighboring agents, but also control updates. From an implementation point of view, it's still a very important issue the for event-triggered control strategies of multi-agent systems with discrete-time dynamics. The event-triggered and self-triggered average consensus for discrete-time multi-agent systems were respectively reported in Chen [43] and Hamada [27]. The event-triggered consensus analysis, which was related to a class of discrete-time stochastic multi-agent systems, was concerned by Ding et al. [44] by using a discrete-time version of the input-to-state stability in probability. The decentralized event-triggered consensus problem was discussed in [55] for discrete-time multi-agent systems linear dynamics. Using the Kronecker product technique and the Lyapunov functional method, Yin et al. [46] considered the event-triggered consensus problem which consist of two kinds of agents differed by their dynamics for a set of discrete-time heterogeneous multi-agent systems. In order to ensure the consensus of heterogeneous multi-agent systems in terms of linear matrix inequality, a sufficient condition was established. In some recent papers, Nedic et al. [56] developed a broadcast-based algorithm, called the subgradient-push, which guided every agent to an optimal value under a standard assumption of subgradient boundedness. The subgradient-push requires neither the knowledge of the number of agents nor the graph sequence to implement. The authors in [47] worked out an event-triggered consensus problem of first-order discrete-time multi-agent systems and firstly proven that the Zeno-like behavior for triggering time sequence is excluded in terms of the discrete-time models. Then, the consensus problem of first-order discrete-time multi-agent systems with time delay via an event-triggered method was considered in Pu et al. [45]. In [45], the authors designed the event-triggered controller with input time delay and the triggering time sequence was determined by some given triggering function. In addition, as far as the theoretical analysis is concerned, the Zeno-like behavior of the trigger function with time delay is excluded for the closed loop system.

1.1. Comparison and contribution

In [56], the authors studied the distributed convex optimization problem of discrete-time multi-agent systems over time-varying directed graphs without event-trigged control. Moreover, the authors in [45,47], considered the consensus problem of first-order discrete-time multi-agent systems, but they did not study the distributed convex optimization problem. Based on the works [45,47,56] , we further consider the convex optimization problem of multi-agent systems via an event-triggered distributed sampling control scheme, where the controllers exchange information through a shared limited communication way over an undirected network topology. On the one hand, there is little existing work concerning with the distributed convex optimization problem of first-order discrete-time multi-agent systems by fully taking advantage of the event-triggered broadcasting communication technique such as to further save the communication cost. On the other hand, our analysis method is novel, which exactly incorporates the distributed optimization technique into the event-triggered control. More precisely, the main contributions of this paper can be summarized as follows: (I) We study the convex optimization problem of first-order discrete-time multi-agent systems by a distributed event-triggered sampling control scheme, where the event-triggered control strategy in this paper can remove unnecessary communications among neighboring agents, resulting in the reduction of computation costs and energy consumptions in practice. (II) A distributed event-triggered control scheme for each agent is designed, which analytically decides the next sampling time instant utilizing the exchange local information. Based on the event-triggered control scheme, a distributed control algorithm is presented, which only employs the agents local information at their latest sampling time instant. (III) We also show the convergence of the algorithm and prove that the event-triggered distributed subgradient algorithm is able to make the whole agents asymptotically converge to an optimal solution.

The rest of this paper is structured as follows. Section 2 introduces some essential concepts and knowledge with regard to the algebraic graph theory, and to formulate the problem of interest. In Section 3, we present the optimization algorithm along with some useful assumptions. Section 4 is devoted to establishing the convergence for the proposed event-triggered distributed subgradient algorithm. The feasibility of the algorithm is demonstrated by applying a numerical example in Section 5. Finally, some concluding remarks and some future directions are presented to close the paper in Section 6.

2. Preliminaries and concepts

In this section, we introduce some essential knowledge with regard to the algebraic graph theory, and formulate the problem of interest. (referring to [49,50,57]).

2.1. Algebraic graph theory

We employ a graph to represent the information exchange between agents. The information exchange between N agents in an information interaction topology can be described as a weighted undirected graph $G = \{V, E, W\}$, where $V = \{1, 2, \ldots, N\}$ and $E \subseteq V \times V$ represent the set of agents with i representing the ith vertex and the undirected set of edges, respectively. The weighted adjacency matrix of G is denoted by $W = [w_{ij}] \in R^{N\times N}$ with nonnegative adjacency elements w_{ij}. Note that the diagonal elements $w_{ii} = 0$ and W is a symmetric matrix. An undirected edge denoted by a pair (j, i) indicates that the information between agent j and agent i can be exchanged. If an edge $(j, i) \in E$, then $w_{ij} > 0$ and agent j is called a neighbor of agent i. We denote $|N_i|$ the neighbor index agent set of agent i and we formulate $|N_i|$ as the number of neighbors of agent i. The Laplacian matrix $L = (l_{ij})_{N\times N}$ of graph G associated with the adjacent matrix W is denoted by $l_{ij} = -w_{ij}$, $i \neq j$; $l_{ii} = \sum_{j=1, j\neq i}^{N} w_{ij}$, which assures that $\sum_{j=1}^{N} l_{ij} = 0$. A path in the graph G from agent j to agent i can be described by a group of head and tail connected edges (j, i_1), (i_1, i_2), … ,(i_m, i) with different agents i_k, $k = 1, 2, \ldots, m$. An undirected graph, which always exists a path between any two distinct agents in the agent set, is defined as an undirected connected graph.

2.2. Notations

Some fairly standard notations throughout the paper are listed in the following. $R^{N\times N}$ and R^N, R refer to the set of $N \times N$ real matrices,