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Finite-time synchronization of multi-layer nonlinear coupled complex networks via intermittent feedback control

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ABSTRACT

This paper addresses the problem of finite-time synchronization for a class of multi-layer nonlinear coupled complex networks via intermittent feedback control. Firstly, based on finite-time stability theory, some novel criteria are given to guarantee that the error system of drive-response systems is still finite-time stable under an inherently discontinuous controller. Then, by proposing two kinds of intermittent feedback control laws, sufficient conditions of finite-time synchronization of two kinds of multi-layer complex networks are derived, respectively. The time delay between different layers is also taken into consideration. Finally, a numerical example is provided to verify the effectiveness of the proposed methods.

1. Introduction

In the past few decades, the synchronization problem of complex networks has attracted more and more attention in practical applications [1–6]. A basic complex network consists of some nodes and links between the nodes, where each node is a dynamic system. Since the problem of synchronization of chaotic systems has been studied in [1], synchronization as a potential engineering application has been applied into secure communication, neural network, biology and information processing [7–11]. Up till now, there are lots of different types of synchronization, for instance, complete synchronization [12], anti-synchronization [13], projective synchronization [14] and cluster synchronization [15,16].

It should be noted that information of different nodes is transmitted based on a shared band-limited digital communication network. Thus, it is interesting to study synchronization of complex networks with delayed coupling. For example, global synchronization of a general linear coupled network has been studied with a time-varying coupling delay in [17]. Then, a developed generalized mixed outer synchronization are also studied with a time-varying coupling delay [18]. In [19], local and global synchronization of complex networks have been studied with a fixed delay. In [20], global exponential synchronization of nonlinear coupled dynamical networks are also considered with a delayed coupling. However, the aforementioned results are based on one or two layers network. Multi-layer networks which have more than two layers can be seen as some sub-networks distributed in different layers. For example, there exists a three-layers network about information transmission in a simple telephone network. Moreover, different transmission delays between different layers should also be taken into account. Therefore, synchronization of multi-layer networks with delayed coupling are more significant.

Different from continuous control methods, intermittent controller is implemented intermittently during a control period. Because of easier implementation and smaller control cost, the problem of synchronization under intermittent control has attracted lots of attention [21–26], since the intermittent control is firstly proposed in [27]. Synchronization with finite time convergence has advantages to enhance the robustness and to overcome the disturbance in practical control and applications [28]. The existing results about finite-time stability and finite-time synchronization have been considered in [29–36]. Therefore, it is very interesting to investigate finite-time synchronization of complex networks via intermittent feedback control. Some related results have been studied in our previous works [37–40], however, the linear coupling is adopted in these works.

In this paper, finite-time synchronization of multi-layer nonlinear coupled complex networks is studied via intermittent feedback control. Firstly, based on finite-time stability theory, some novel criteria are given to guarantee that the nonlinear system is still finite-time stable. Then, by proposing two kinds of intermittent feedback controllers, sufficient conditions of finite-time synchronization of two complex networks are derived. The main contributions of this paper include: i) some novel criteria are given to guarantee finite-time synchronization of the error system of the drive-response systems under an intermittent controller; ii) then, based on these presented criteria, finite-time

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synchronization of two kinds of multi-layer nonlinear coupled networks is studied via periodically intermittent feedback control and aperiodically intermittent feedback control, respectively. The time delay between different layers is also taken into consideration. The corresponding sufficient conditions are also given to guarantee that the error system is finite-time stable.

This paper is organized as follows. In Section 2, some definitions of finite-time stability and some novel finite-time criteria are given. In Section 3, by proposing two kinds of intermittent feedback controllers, sufficient conditions of finite-time synchronization of delayed complex networks are derived respectively. Section 4 provides an example to illustrate the validity of the proposed design methods. Finally, this paper is concluded in Section 5.

2. Preliminaris

Let $\mathbb{R}^n$ denote n-dimension real space and $\mathbb{R}^+$ denote 1-dimension positive real space. For any $x \in \mathbb{R}^n$, let $\| x \| = (x^T x)^{1/2}$. For a matrix $P \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the largest and the smallest eigenvalues of the symmetric matrix P, respectively.

Consider the following master system (drive system):

$$\dot{x}(t) = \phi(x(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $x(0) = x_0$, $\phi(\cdot)$: $\mathcal{D} \to \mathbb{R}^n$ is continuous on an open neighborhood $\mathcal{D}$ of the origin $x(t) = 0$ with $\phi(0) = 0$.

Definition 1. [41] The zero solution of (1) is finite-time convergent if there is an open neighborhood $\mathcal{U} \subset \mathcal{D}$ of the origin and a function $\mathcal{T}$: $\mathcal{U}\backslash\{0\} \to (0, \infty)$, such that $\forall\, x_0 \in \mathcal{U}$, the solution $\psi(t, x_0)$ of system (1) is defined and $\psi(t, x_0) \in \mathcal{U}\{0\}$ for $t \in [0, \mathcal{T}(x_0))$, and $\lim_{t\to\mathcal{T}(x_0)}\psi(t, x_0) = 0$. Then, $\psi(t, x_0)$ is called the settling time. If the zero solution of system (1) is finite-time convergent, the set of point x_0 such that $\psi(t, x_0) \to 0$ is called the domain of attraction of the solution.

Definition 2 (*[41]*). The zero solution of (1) is finite-time stable if it is Lyapunov stable and finite-time convergent. When, $\mathcal{U} = \mathcal{D} = \mathbb{R}^n$, the zero solution is said to be globally finite-time stable.Consider the following slave system (response system):

$$\dot{y}(t) = \varphi(y(t), u(t)), \tag{2}$$

where $y(t) \in \mathbb{R}^n$, $y(0) = y_0$, $u(t) \in \mathbb{R}^q$ is the controller, $u(0) = u_0$, $\varphi(\cdot)$: $\mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n$ is continuous. Denote the solutions of (1) and (2) as $x(t, x_0)$ and $y(t, y_0, u_0)$, respectively. For the notational simplicity, we denote $x(t, x_0)$ simply by $x(t)$, and $y(t, y_0, u_0)$ by $y(t)$. Next, we give the definition of finite-time synchronization of systems (1) and (2).

Definition 3. Systems (1) and (2) are said to be synchronization in finite time if there exists an open neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin such that $e_0 = y_0 - x_0 \in \mathcal{U}$ and a function $\mathcal{T}_1$: $\mathcal{U}\backslash\{0\} \to (0, +\infty)$ and

$$\lim_{t\to\mathcal{T}_1(e_0)} \| e(t)\| \to 0,\ \| e(t)\| = 0,\ \forall\ t > \mathcal{T}_1(e_0),$$

where $e(t) = y(t) - x(t)$ denotes the synchronization error of systems (1) and (2).

A continuous controller is designed in the form of $u(t) = \mathcal{F}(e(t)), \forall\ t \in [t_0, +\infty)$. If there exists a Lyapunov function $V(e(t))$ defined on a neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin such that $\dot{V}(e(t)) \le -\alpha V^\eta(e(t))$, where $\alpha > 0$, $0 < \eta < 1$, from [42] and Definition 1, the error system (2)-(1) is synchronized in finite time. Based on our previous work [37], a new controller is proposed as follows:

$$\begin{cases} u(t) = 0, & t_0 + kT \le t < t_0 + (k + h_1)T, \\ u(t) = \mathcal{F}(e(t)), & t_0 + (k + h_1)T \le t < t_0 + (k + h_2)T, \\ u(t) = 0, & t_0 + (k + h_2)T \le t < t_0 + (k + 1)T, \end{cases} \tag{3}$$

where $0 \le h_1 < h_2 \le 1$, $T > 0$ is the control period, $h_2 - h_1$ is the control rate and $k \ge 0$ is a nonnegative integer. Now, sufficient conditions are given to guarantee that the error systems (1)–(2) is synchronized in finite time via the controller (3).

Theorem 1. Consider systems (2) and (1) with controller (3), if there is a Lyapunov function $V(e(t))$ defined on a neighborhood $\mathcal{U} \subset \mathbb{R}^n$ of the origin such that

$$\begin{cases} \dot{V}(e(t)) \le 0, & t_0 + kT \le t < t_0 + (k + h_1)T, \\ \dot{V}(e(t)) \le -\alpha V^\eta(e(t)), & t_0 + (k + h_1)T \le t < t_0 + (k + h_2)T, \\ \dot{V}(e(t)) \le 0, & t_0 + (k + h_2)T \le t < t_0 + (k + 1)T, \end{cases} \tag{4}$$

hold, where $0 \le h_1 < h_2 \le 1$, $\alpha > 0$, $0 < \eta < 1$, then, the error system (2)-(1) is synchronized in finite-time. In addition, for any given t_0, the following inequality holds:

$$V^{1-\eta}(e(t)) \le V^{1-\eta}(e_0) - \alpha(1 - \eta)(h_2 - h_1)(t - t_0 - h_1T), \quad t_0 \le t \le T', \tag{5}$$

and $V(e(t)) \equiv 0$, $\quad\forall\ t > T'$, where $T' = \frac{V^{1-\eta}(e_0)}{\alpha(1-\eta)(h_2-h_1)} + t_0 + h_1T$ denotes the settling time.

Proof. The proof is based on a recursive approach and the following auxiliary function

$$H(t) = V^{1-\eta}(t) - M + \alpha(1 - \eta)(h_2 - h_1)t, \tag{6}$$

where $M = V^{1-\eta}(e_0) + \alpha(1 - \eta)(h_2 - h_1)(t_0 + h_1T)$. It is also easy to obtain that $H(t_0) < 0$. For simplicity, we denote $V(e(t))$ as $V(t)$.

Step 1: For any $t \in [t_0, t_0 + h_1T)$, we have

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0).$$

Then, we can obtain

$$H(t) \le V^{1-\eta}(t_0) - M + \alpha(1 - \eta)(h_2 - h_1)t < 0.$$

For any $t \in [t_0 + h_1T, t_0 + h_2T)$, we have

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0 + h_1T) - \alpha(1 - \eta)(t - t_0 - h_1T) \quad \le V^{1-\eta}(t_0) - \alpha(1 - \eta)t + \alpha(1 - \eta)(t_0 + h_1T).$$

Then,

$$\begin{aligned} H(t) &\le V^{1-\eta}(t_0) - \alpha(1 - \eta)t + \alpha(1 - \eta)(t_0 + h_1T) - M + \alpha(1 - \eta)(h_2 - h_1) \\ &t \le -\alpha(1 - \eta)t + \alpha(1 - \eta)(t_0 + h_1T) - \alpha(1 - \eta)(h_2 - h_1)(t_0 + h_1T) \\ &+ \alpha(1 - \eta)(h_2 - h_1)t \le \alpha(1 - \eta)(1 - (h_2 - h_1))(t_0 + h_1T - t) \le 0. \end{aligned} \tag{7}$$

For any $t \in [t_0 + h_2T, t_0 + T)$, we have

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0 + h_2T) \le V^{1-\eta}(t_0) - \alpha(1 - \eta)(t_0 + h_2T) + \alpha(1 - \eta)(t_0 + h_1T).$$

Then,

$$\begin{aligned} H(t) &\le V^{1-\eta}(t_0) - \alpha(1 - \eta)(t_0 + h_2T) + \alpha(1 - \eta)(t_0 + h_1T) - M \\ &+ \alpha(1 - \eta)(h_2 - h_1)t \le \alpha(1 - \eta)(h_2 - h_1)(t - t_0 - h_1T - T) < 0. \end{aligned} \tag{8}$$

Step 2: For any $t \in [t_0 + T, t_0 + (1 + h_1)T)$, we have

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0 + T) \le V^{1-\eta}(t_0) - \alpha(1 - \eta)(t_0 + h_2T) + \alpha(1 - \eta)(t_0 + h_1T).$$

Then,

$$\begin{aligned} H(t) &\le V^{1-\eta}(t_0) - \alpha(1 - \eta)(t_0 + h_2T) + \alpha(1 - \eta)(t_0 + h_1T) - M \\ &+ \alpha(1 - \eta)(h_2 - h_1)t \le \alpha(1 - \eta)(h_2 - h_1)(t - t_0 - h_1T - T) < 0. \end{aligned} \tag{9}$$

For any $t \in [t_0 + (1 + h_1)T, t_0 + (1 + h_2)T)$, we have

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0 + T + h_1T) - \alpha(1 - \eta)(t - t_0 - h_1T - T) \le V^{1-\eta}(t_0) - \alpha(1 - \eta)t + \alpha(1 - \eta)[t_0 + (h_1 + 1)T - (h_2 - h_1)T].$$