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Nearly optimal sliding mode fault-tolerant control for affine nonlinear systems with state constraints

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ABSTRACT

In this paper, the problem of fault-tolerant control for a class of affine nonlinear systems subject to state constraints is investigated. Firstly, the nearly optimal controller design for the constrained nominal system is considered. By introducing proper slack variables, the nonlinear optimal control problem with state constraints is transformed into an unconstrained one for an augmented nonlinear system. Then, an adaptive weight update algorithm based on neural network (NN) is given to solve the Hamilton–Jacobi–Bellman (HJB) equation, which can generate an approximated optimal control policy for the augmented system. In order to achieve the fault-tolerant control objective, an NN observer with novel adaptive laws is constructed to identify the actuator faults. An integral sliding mode fault-tolerant control scheme is employed to guarantee the stability of the constrained faulty system, which is designed using the fault estimations and the obtained nominal nearly optimal control policy. Finally, simulations are presented to illustrate the effectiveness of the proposed method.

1. Introduction

Increasing requirements for high performance make the control systems more and more complex, which means that the system components are more likely to suffer failures (actuator faults, sensor faults and even the system faults). Once the faults occur, the control systems may end up with the performance degradation and even instability if the faults are not well handled. Therefore, fault tolerant control (FTC) [1–3], in which the aim is to guarantee the acceptable performance and stability for fault-free or faulty systems, has received considerable attention in the past decades. Among the different schemes, FTC can mainly be classified into two categories: passive and active approaches. Robust control theory is usually used in the passive FTC approaches [4–6], which is easy to design and implement but has limited fault-tolerant ability. By contrast, the active FTC approaches can compensate for faults and modify the control policy adaptively based on the fault detection and estimation schemes (see [7–9]). From the above related literature, it is found that most of the FTC methods are considered for linear systems. As a matter of fact, the nonlinearity exists commonly in most of the practical control systems [10]. Therefore, it is more desirable to design FTC methods for nonlinear systems.

Actually, all the practical systems are subject to some kinds of physical constraints more or less [11], which are often known as input and state constraints. Violation of these constraints may result in performance degradation or even safety hazard. The existence of these constraints also makes it more difficult to design suitable control policies to stabilize systems and achieve the desirable performance. Therefore, the control problems with constraints have received more and more attention in the past years. For example, as an effective methodology, model predictive control (MPC) has been widely adopted to handle both constraints and performance issues in a finite horizon optimal control framework [12,13]. However, the MPC methods are almost numerical and often have heavily computational complexity. Barrier Lyapunov function approach is another well-known method to deal with the constrained control problems [14–16], in which the backstepping control method is often used for nonlinear systems with special triangular structures. Meanwhile, optimal control is also a suitable methodology to solve the constrained control problems. In [17,18], the penalty functions were introduced to approximately transform the constrained optimal problems into unconstrained ones. A novel transformation technique was proposed in [19] to transform an optimal control problem with a scalar state inequality constraint into an unconstrained problem. To overcome the singular arcs problem in [19], a special quadratic function was added to the performance index in [20], which is suitable for finite horizon optimal control problems with a scalar control variable and a scalar state inequality constraint. Moreover, it is still difficult to

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solve the transformed optimal control problems for nonlinear systems.

The main obstacle to solve the nonlinear optimal control problem is that the HJB equation is difficult or impossible to solve [21]. Although the dynamic programming method [22] was proposed to overcome the obstacle, it is often implemented off-line and vulnerable to the influence of "curse of dimensionality". Recently, using NN to approximately solve the nonlinear optimal control problems has been considered widely [23,24], which is derived from the adaptive dynamic programming (ADP) method proposed in [25]. Based on policy iteration (PI) [26], an adaptive actor-critic structure was presented in [27] to solve the infinite horizon optimal control problem for nonlinear systems with known dynamics. Bian et al. [28] proposed a novel ADP method to solve the optimal control problem for unknown nonaffine nonlinear systems. In [29,30], the nonlinear optimal control and optimal tracking problems with input constraints were considered. Although the ADP methods have been investigated widely to handle the nonlinear control problem [31–35], it has hardly been employed to handle the FTC problem of nonlinear systems with state constraints.

In order to compensate the effect of faults, various approaches have been proposed. In addition to H_∞ control, another effective approach to handle the FTC problem is the sliding mode control (SMC) method [36–38]. To avoid the reaching phase problem in the classical SMC methods, the integral SMC (ISMC) was proposed in [39,40]. For nonlinear systems, the ISMC is not easy to implement because the controller stabilizing the nominal nonlinear dynamics is difficult to design without specific assumptions for system structures. In [41,42], nonlinear fault tolerant control methods were proposed based on the integral sliding mode control allocation scheme, while it was assumed that there exists a known controller guaranteeing the stability of the nominal system.

Motivated by the aforementioned analysis, the current study is aimed at designing a nearly optimal sliding mode FTC scheme for affine nonlinear systems with state constraints. The main contributions of this paper are summarized as follows:

1. To avoid the appearance of singular arcs in the transformation technique of [19] and modifying the finite horizon performance index in [20], new slack functions are introduced to transform the optimal control problem with state inequality constraints into an unconstrained one which can be solved based on the ADP method.
2. An NN-based observer and an adaptive sliding mode control method are combined to compensate the faults and guarantee the nearly optimal performance of the sliding mode dynamics.

The rest of this paper is organized as follows: The problem is formulated in Section 2. In Section 3, the design method of the nearly optimal control policy for the nominal constrained nonlinear system is proposed. It is followed by designing the fault identification and FTC control strategy in Section 4. Then, simulation results are given in Section 5 to verify the effectiveness of the proposed method. Finally, Section 6 draws the conclusion.

Notations: X^T denotes the transpose of a matrix X, I represents the identity matrix with appropriate dimension. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $diag(X_1, X_2, \ldots, X_n)$ denotes a block diagonal matrix with matrices $X_1, X_2, \ldots, X_n$ on its main diagonal. We define the norms of a vector $a = (a_1, a_2, \ldots, a_k)^T$ as $|a| = \sum_{i=1}^{k} |a_i|$ and $\|a\| = \sqrt{\sum_{i=1}^{k} a_i^2}$. $\lambda_{min}(X)$ is defined as the minimum eigenvalue of matrix X. tr(X) denotes the trace of the matrix X.

2. System description and preliminaries

Consider a class of continuous-time nonlinear systems:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the measurable system state, $u(t) \in \mathbb{R}^m$ is the control input, $f(x(t)) \in \mathbb{R}^n$ and $g(x(t)) \in \mathbb{R}^{n\times m}$ are differentiable and Lipschitz continuous with $f(0) = 0$. To simplify writing, $x(t)$ is abbreviated to x in some subsequent formulas.

The state vector x is considered to be constrained with the following way:

$$L_j(x) < 0, \quad j = 1, \ldots, l, \tag{2}$$

where $L_j(x)$ is the p_jth order differentiable function, and it is assumed that u appears firstly in the p_jth derivative of $L_j(x)$. It is assumed that the set $\Phi_x = \{x | L_j(x) < 0,\ j = 1, \ldots, l\}$ contains the origin.

It is known that actuators may become faulty in practical application. In this paper, the system model with actuator faults is given as follows:

$$\dot{x}(t) = f(x) + g(x)[u(t) + \varpi(t-T)\Delta(x)], \tag{3}$$

where

$$\varpi(t-T) = \begin{cases} 0, & t < T \\ 1, & \text{otherwise}, \end{cases} \tag{4}$$

which means that the abrupt faults are considered.

It is known that NNs have good capabilities of nonlinear function approximation. Considering the Weierstrass higher-order approximation theorem [43] and the results of [44], a three-layer NN is sufficient to approximate nonlinear systems with any degree of nonlinearity on a compact set. Thus, the nonlinear function $\Gamma(x) \in \mathbb{R}^{l_0}$ can be represented as

$$\Gamma(x) = W_0 \sigma(V_0 x) + \varepsilon(x), \tag{5}$$

where $W_0 \in \mathbb{R}^{l_0 \times N_0}$ and $V_0 \in \mathbb{R}^{N_0 \times n}$ are ideal weights, N_0 is the number of the neurons. $\sigma(\cdot)$ and $\varepsilon(x) \in \mathbb{R}^{l_0}$ are the nonlinear activation function and the function approximation error, respectively.

The objective is to design a suitable control $u = u_0$ to stabilize the nominal system (1) without violating the constraints in (2) and to optimize the performance index

$$V(x) = \int_t^\infty [Q(x(\tau)) + u^T R u] d\tau, \tag{6}$$

where $Q(x)$: $\mathbb{R}^n \to \mathbb{R}$ is a continuous positive definite function that satisfies $q_{min}\|x\| \le Q(x) \le q_{max}\|x\|$, where q_{min} and q_{max} are proper positive constants. $R > 0$ are suitable symmetric matrices. Meanwhile, a fault identification scheme is designed. As soon as actuator faults are detected, the fault-tolerant controller $u = u_0 + u_1$ is activated to guarantee the stability of the faulty system with the constraints (2) and the nearly optimal performance of the sliding mode dynamics.

To ensure the control objective, the following assumptions are needed.

Assumption 1. The ideal NN weights W_0 and V_0 are bounded, that is $\| W_0 \| \le \bar{W}_0, \| V_0 \| \le \bar{V}_0$. The NN activation function and the approximation error are bounded, that is $\| \sigma(x) \| < \bar{\sigma}$ and $\| \varepsilon(x) \| < \bar{\varepsilon}$.

Assumption 2. $g(x)$ is full column rank and is bounded for any x. There exists a positive constant $\bar{g}$ such that $\| g(x) \| \le \bar{g}$. There exists a proper bounded function $q(x)$ so that $q(x)g(x)$ is invertible for any x.

Remark 1. Assumption 1 is commonly used in the literature based