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Distributed adaptive consensus tracking control of higher-order nonlinear strict-feedback multi-agent systems using neural networks ☆

Gang Wang [a], Chaoli Wang [a,*], Lin Li [a], Qinghui Du [b]

[a] University of Shanghai for Science and Technology, Shanghai 200093, China
[b] Department of Mathematics, Luoyang Normal University, Luoyang 471022, China

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ABSTRACT

This paper considers the output consensus problem of tracking a desired trajectory for a group of higher-order nonlinear strict-feedback multi-agent systems over directed communication topologies. Only a subset of the agents is given direct access to the desired trajectory information. A distributed adaptive consensus protocol driving all agents to track the trajectory is presented using the backstepping technique and neural networks. The Lyapunov theory is applied to guarantee that all signals in the closed-loop system are uniformly ultimately bounded and that all agents' outputs synchronize to the desired trajectory with bounded residual errors. Compared with prior work, the dynamics of each agent discussed here is more general and does not require the assumption "linearity in the unknown parameters" or the matching condition. Moreover, the bounded residual errors can be reduced as small as desired by appropriately choosing design parameters. Simulation results are included to demonstrate the effectiveness of the proposed methods.

1. Introduction

In recent years, research on multi-agent systems has attracted a considerable amount of attention due to its potential applications in sensor networks, monitoring and surveillance, and unmanned-air-vehicle formations [1]. One major topic of interest is the consensus control design of multi-agent systems. The current consensus methods can be roughly categorized into two classes, namely, leaderless consensus and leader-following consensus (i.e., distributed tracking) [2]. Most of the results on consensus in the existing literature are presented for linear multi-agent systems [3–5]. However, it is well known that, in engineering, many systems have more complicated dynamics and need to be modeled by nonlinear dynamics, such as nonholonomic mobile robots [6] or robotic manipulators [7]. Therefore, compared with linear multi-agent systems, consensus control of nonlinear multi-agent systems is more valuable for practical application, and it is also more challenging due to the complexity of its system dynamics.

Recently, distributed consensus control approaches [8–10] have been developed for nonlinear multi-agent systems and have received increasing attention. For a class of first-order nonlinear multi-agent systems, Hou et al. [8] presented a distributed control method to solve the consensus problem by employing an adaptive neural networks (NN) scheme. Chen et al. [9] extended the result [8] to first-order time-delay multi-agent systems with nonlinear dynamics and an undirected communication topology. A Lyapunov–Krasovskii functional is constructed to compensate for the uncertainties of unknown time delays. Previously, [10] investigated the output synchronization of higher-order multi-agent systems with parameter uncertainty. One common feature in the above results [8–10] is that the proposed control methods can only solve the leaderless consensus problem. In real-life applications, the multi-agent systems usually need to track a leader or a desired trajectory rather than rendezvousing to a common value (see, for example, flocks, herds and schools [11]). Therefore, the leader-following consensus of multi-agent systems is closer to real-life applications than leaderless consensus.

As an alternative, a series of papers dealt with the leader-following consensus problem of nonlinear multi-agent systems. Distributed adaptive control algorithms [12–14] were proposed for first-order nonlinear multi-agent systems, in which the neural networks were used to actively compensate for the agent's uncertain dynamics. Furthermore, the online computation burden has been taken into account in [14]. In [15], Das and Lewis studied the leader-following consensus problem for multi-agent systems with second-order integrator dynamics over a directed connected graph. The results [13,15] were further generalized to general higher-order nonlinear systems in the Brunovsky form in [16,17]. It is worth noting that the unknown nonlinear dynamics was assumed to be in the range space of the control

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* Corresponding author.
E-mail address: clclwang@126.com (C. Wang).

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input. That is, the uncertainties must satisfy the matching condition in [13,15–17], which may limit their applications. With further development, Wang et al. [18] proposed a backstepping-based distributed adaptive controller to achieve output consensus tracking with strict-feedback multi-agent systems. Moreover, the design strategy is successfully applied to solve a formation control problem for multiple nonholonomic mobile robots. Nevertheless, the control approach in [18] suffers from the assumption that the unknown nonlinear dynamics must satisfy the condition "linearity in the unknown parameters". In real world applications, such an assumption cannot always hold. This gap in knowledge has been highlighted in many papers, such as [8,12].

Motivated by the previous discussion, in this paper, we intend to design a distributed tracking control method for general higher-order nonlinear strict-feedback systems over a directed communication topology. Suppose that only a subset of the agents is direct given access to the information of the desired trajectory. Using the backstepping technique and the approximation technique of NN, a distributed adaptive consensus control law is constructed for each agent based on only the relative state between itself and its neighbors. Meanwhile, the projection algorithm is applied to ensure that the estimated parameters remain in some known bounded sets. It is shown via Lyapunov theory that the proposed adaptive method can guarantee that all signals in the closed-loop system are uniformly ultimately bounded and that all agents' outputs synchronize to the desired trajectory with bounded residual errors. Moreover, the errors can be reduced as small as desired by appropriately choosing design parameters.

Compared with the aforementioned work, the main contributions of this paper consist of the following aspects:

1. Although the work in [12–14] addressed distributed tracking issues of the multi-agent systems, the results are only suitable for first-order multi-agent systems. However, in this paper, we focus on the consensus problem of higher-order multi-agent systems, which are more general and include first-order systems [12–14] as a special case.
2. In contrast to the leader-following consensus approaches for second- or higher-order multi-agent systems [15–17], our proposed algorithm does not require the matching condition but allows for more general higher-order nonlinear systems with mismatched uncertainties. Additionally, the results in [15–17] can only guarantee that tracking errors are uniformly ultimately bounded, while our proposed approach can make the tracking errors as small as possible by appropriately choosing design parameters.
3. Although the work in [18] discussed the distributed adaptive control problem for strict-feedback systems, its control approach suffers from the assumption "linearity in the unknown parameters". Moreover, each agent has to know its neighbors basis functions (see the virtual controllers (17) and (25) in [18]). Contrarily, the proposed approach in this paper does not require the assumption "linearity in the unknown parameters", and for each agent, only the relative state between itself and its neighbors is needed in the controller design.

Throughout this paper, the following notations are used. $\underline{1}_N$ is the N-vector of ones; $\|\cdot\|$ is the Euclidean norm of a vector; $\| H \|_F = \sqrt{\mathrm{tr}\{H^T H\}}$ is the Frobenius norm of matrix H with $\mathrm{tr}(\cdot)$ being the trace; $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ are the maximum singular value and the minimum singular value of a matrix, respectively.

2. Problem statement

2.1. Basic graph theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a directed graph to model communication among N agents, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of nodes corresponding to each agent and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. $(i, j) \in \mathcal{E}$ denotes that agent j can obtain information from agent i, but not necessarily vice versa for a directed graph. In this paper, self edges are not allowed, i.e., $(i, i) \notin \mathcal{E}$. The neighbors of agent i is $N_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. Denote the adjacency matrix by $A = [a_{ij}] \in R^{N\times N}$ with $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. Define the in-degree matrix $D = \mathrm{diag}\{d_1, \ldots, d_N\}$ and the Laplacian matrix $L = D - A$, where $d_i = \sum_{j=1}^{N} a_{ij}$. A direct path from agent i to agent j is a sequence of successive edges in the form $\{(i, l), (l, m), \ldots, (k, j)\}$. A directed graph $\mathcal{G}$ has a spanning tree if there exists an agent i called the root such that there is a direct path from agent i to every other agent in the graph.

2.2. Problem formulation

Consider a group of $N (N \geq 2)$ nonlinear strict-feedback multi-agent systems. The dynamics of the ith $(i = 1, \ldots, N)$ agent can be modeled as

$$\dot{x}_{i,m} = x_{i,m+1} + f_{i,m}(\bar{x}_{i,m}), \quad m = 1, \ldots, n-1$$
$$\dot{x}_{i,n} = u_i + f_{i,n}(\bar{x}_i),$$
$$y_i = x_{i,1}, \tag{1}$$

where $\bar{x}_i = [x_{i,1}, \ldots, x_{i,n}]^T \in R^n$, $u_i \in R$, and $y_i \in R$ are the state vector, the control input and the output of agent i, respectively; $\bar{x}_{i,m} = [x_{i,1}, \ldots, x_{i,m}]^T$; $f_{i,m}(\cdot) \in R$, $m = 1, \ldots, n$, is the unknown smooth nonlinear function that contains both parametric and nonparametric uncertainties. The control objective in this paper is to design the control input u_i to make y_i converge to the desired trajectory y_d with a small error, while the other signals remain bounded.

Remark 1. Obviously, when $f_{i,m} = 0$, $m = 1, \ldots, n-1$, the agent model in (1) reduces to first-order nonlinear multi-agent systems [12,13], second-order multi-agent systems [15], or higher-order multi-agent systems [16,19]. It is also easy to see that model (1) can take nonlinear subsystems with intrinsic mismatched unknown parameters [18] as a special case. Therefore, the agent model discussed in this paper is more general.

The desired trajectory y_d can be expressed by a linear combination of r basis functions, that is,

$$y_d(t) = \sum_{l=1}^{r} f_{d,l}(t)\phi_{d,l} + c_d = f_d(t)^T \phi_d + c_d, \tag{2}$$

where $f_d(t) = [f_{d,1}(t), \ldots, f_{d,r}(t)]^T \in R^r$ is the vector of basis functions that are available to all of the N agents. However, $\phi_d = [\phi_{d,1}, \ldots, \phi_{d,r}]^T \in R^r$ and $c_d \in R$ are constant parameters that are known only to a subset of agents.

Assumption 1 (*[18]*). The basis function $f_{d,m}(t)$ $(\forall\, m = 1, \ldots, r)$ satisfies that $f_{d,m}^{(l)}$ $(\forall\, l = 0, \ldots, n)$ is bounded, where the notation $f_{d,m}^{(l)}$ denotes the lth derivative of $f_{d,m}$ (i.e., $f_{d,m}^{(l)} = d^l f_{d,m}/dt^l$; $f_{d,m}^{(0)} = f_{d,m}$). In addition, $f_{d,m}^{(l)}$ is supposed to be known to all agents in the group.

Assumption 2. There exist positive constants Φ_M and F_M such that $\| \phi_d \| \leq \Phi_M$ and $\| \dot{f}_d(t) \| \leq F_M$.

Remark 2. Notably, the trajectory given in (2) is a commonly employed expression that has appeared in the relevant literature, such as [18,20,21]. As we know, a function can be represented or approximated as a linear combination of a set of prescribed basis functions in a function space. For example, y_d can be transformed into the Fourier series within a time interval $[0, T]$ as $y_d(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(\omega_n t) + b_n \sin(\omega_n t)]$, where

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