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Linear optimal filtering for time-delay networked systems subject to missing measurements with individual occurrence probability

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ABSTRACT

In this paper, we discuss the problem of optimal filter design for a class of networked stochastic systems subject to state delay and missing measurements. Both the random perturbations and the missing measurements are addressed in the system model, where the random perturbations are characterized by the multiplicative noises and the addressed phenomena of the missing measurements are modeled by a series of mutually independent Bernoulli random variables with individual occurrence probability. In view of the innovative analysis approach and the recursive projection formula, we design an optimal filter for networked systems with multiplicative noises and missing measurements such that the filtering error is minimized in mean square error sense. The main advantage of the proposed result lies in its recursive form applicable for online computations. In addition, we can see that the filter parameter can be obtained by solving some recursive equations. Finally, we give a numerical simulation example to illustrate the effectiveness of the filtering method proposed in this paper.

1. Introduction

The past few decades have seen a surge of research attention on the filtering problems for complex dynamical systems due to their extensive applications in a variety of practical fields such as communication, guidance, image processing, navigation and control of vehicles, target tracking systems and econometrics [3,1,2,4–7,9,8,10]. So far, many important papers have been published on general filtering problems with many performance requirements, see e.g. [11,12,2,13–15,17,16,18]. As is well known, the classical Kalman filtering algorithm, which has been proposed in [13] by applying the innovative analysis approach and the projection theory, can be regarded as an linear optimal filter in the sense of least mean square for linear stochastic systems. Subsequently, many important results have been reported based on the simple and convenient method of the traditional Kalman filter. For example, the optimal filtering problem has been studied in [14] for the networked systems contaminated with multiple fading measurements, random parameter matrices as well as correlated noises, and new filtering algorithm has been given which is applicable for the online implementation.

Note that the time delays exist commonly in many practical systems. Hence, many results have been reported to address the time delay so as to improve the system performance by proposing many effective control/filtering schemes, see e.g. [19–26]. To name a few, the linear optimal state estimator has been constructed in [27] for a class of discrete systems with *random* measurement delays by applying the state augmentation approach. In contrast, without applying the state augmentation approach, an optimal filter has been designed in [28] for a class of linear discrete stochastic systems with state delay based on the recursive projection formula. It is easily known that the computational burden in [28] is less than using the state augmentation approach. Based on the method in [28], the optimal filtering problem has been researched in [29] for linear discrete stochastic systems subject to the state delay. In [30], the optimal Kalman filtering problem has been addressed for a class of stochastic systems with randomly occurring sensor delays and state delay. Compared with the existing results, the developed filtering algorithm in [30] can better tackle the random sensor delays occurring in engineering practice in a more effective way especially for complex systems with individual delay rate. Moreover, in [31], a new filtering compensation scheme has been given for nonlinear stochastic systems subject to random one-step sensor delay, where both the variance constraint and gain-constraint have been addressed in a same framework.

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During the past two decades, the so-called networked systems have been received considerable attention [32–36], and a timely review has been given in [34] where the recent advances of the estimation problems for networked systems have been reported. Accordingly, great effort has been made to handle the network-induced problems such as the missing measurements, see e.g. [37–41]. By using the state augmentation method, the linear minimum variance estimation problem has been discussed in [42] for a class of stochastic systems with finite random measurement delays and consecutive packet dropouts. Recently, based on the method of the traditional Kalman filter, a quantized recursive filtering method has been proposed in [43] for a class of nonlinear systems with multiplicative noises and missing measurements. Very recently, in [39], the randomly occurring faults and probabilistic fading channels have been addressed in a unified framework, where a new H_∞ estimation algorithm of randomly occurring faults has been developed by using the recursive linear matrix inequality approach. However, to the best of authors' knowledge, the linear optimal filtering problem has not been thoroughly studied for discrete-time state delay networked systems with multiple missing measurements which constitutes our current research motivation.

Based on the above discussions, we can see that it is necessary to deal with the optimal filtering problem for state delay stochastic systems subject to multiple missing measurements. In this paper, both random perturbations and multiple missing measurements are considered in the addressed system model. Here, the random perturbations are characterized by the multiplicative noises. The phenomena of missing measurements occur in a random way and are described by a set of mutually independent Bernoulli random variables with known occurrence probabilities. By resorting to the recursive projection formula and the innovative analysis approach, a new recursive filtering algorithm is given such that, for both the missing measurements and the multiplicative noises, the obtained filtering error is minimized in the least mean square error sense. At last, we provide a numerical example to show the usefulness of the main results. The main contributions of this paper can be summarized as follows: (1) the state delay, the multiple missing measurements and the multiplicative noises are addressed in a unified framework for the optimal filter design problem and (2) a novel optimal filtering scheme is presented for the addressed networked stochastic systems based on the innovative analysis approach and the recursive projection formula.

Notation. The notations used throughout the paper are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. For a matrix P, P^T represents its transpose. $\mathbb{E}\{x\}$ is the expectation of a random variable x. $\text{diag}\{P_1, P_2, \ldots, P_N\}$ stands for a diagonal matrix with elements $P_1, P_2, \ldots, P_N$ in the diagonal. I and 0 stand for an identity matrix and a zero matrix with compatible dimensions, respectively. The Hadamard product is defined as $[T \circ S]_{p\times p} = [t_{ij} \times s_{ij}]_{p\times p}$.

2. Problem formulation

Consider the following discrete networked systems with multiplicative noises and multiple missing measurements:

$$x_{k+1} = A_k x_k + A_{k-d} x_{k-d} + B_k \omega_k \tag{1}$$

$$z_k^i = \lambda_k^i \left(C_k^i + C_{s,k}^i \eta_k \right) x_k + \nu_k^i \tag{2}$$

where $i = 1, 2, \ldots, N$, $x_k \in \mathbb{R}^n$ is the time-invariant state vector, d is the state delay, $z_k^i \in \mathbb{R}^m$ is the i-th output. $\omega_k \in \mathbb{R}^r$ is the zero-mean process noise with covariance $Q_k \geq 0$, $\nu_k^i \in \mathbb{R}^m$ is the zero-mean measurement noise with covariance $R_k^i > 0$. η_k is zero mean multiplicative noise with unity covariance. A_k, A_{k-d}, B_k, C_k^i and $C_{s,k}^i$ are known matrices. The random variables λ_k^i describing the phenomenon of missing measurements obey the Bernoulli probabilistic distribution satisfying:

$$P\left\{ \lambda_k^i = 1 \right\} = \mathbb{E}\left\{ \lambda_k^i \right\} = \alpha_i,$$
$$P\left\{ \lambda_k^i = 0 \right\} = 1 - \mathbb{E}\left\{ \lambda_k^i \right\} = 1 - \alpha_i$$

where $\alpha_i \in [0, 1]$ $(i = 1, 2, \ldots, N)$ are known scalars. In the sequel, we make an assumption that ω_k, $\nu_k{}^i$, η_k and $\lambda_k{}^i$ are mutually independent.

Remark 1. In the model (2), the random perturbations existing in the observation equation are characterized by the multiplicative noises η_k. Moreover, if the random variables $\lambda_k^i = 1$, we have $z_k^i = (C_k^i + + C_{s,k}^i \eta_k) x_k + \nu_k^i$, it means that the i-th sensor receives the measurement data successfully at sampling instant k. If $\lambda_k^i = 0$, $z_k^i = \nu_k^i$, it stands for that the i-th sensor receives the noises only at instant k, i.e., the phenomenon of missing measurements occurs.

Setting

$$z_k = \left[(z_k^1)^T \;\; (z_k^2)^T \;\; \cdots \;\; (z_k^N)^T \right]^T,$$
$$C_k = \left[(C_k^1)^T \;\; (C_k^2)^T \;\; \cdots \;\; (C_k^N)^T \right]^T,$$
$$C_{s,k} = \left[(C_{s,k}^1)^T \;\; (C_{s,k}^2)^T \;\; \cdots \;\; (C_{s,k}^N)^T \right]^T,$$
$$\nu_k = \left[(\nu_k^1)^T \;\; (\nu_k^2)^T \;\; \cdots \;\; (\nu_k^N)^T \right]^T,$$

and $\Lambda_k = \text{diag}\{\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^N\}$, the system (1)–(2) can be rewritten as follows:

$$x_{k+1} = A_k x_k + A_{k-d} x_{k-d} + B_k \omega_k \tag{3}$$

$$z_k = \Lambda_k \left(C_k + C_{s,k} \eta_k \right) x_k + \nu_k \tag{4}$$

It is not difficult to obtain the following fact:

$$\Lambda = \mathbb{E}\left\{ \Lambda_k \right\} = \text{diag}\left\{ \alpha_1, \alpha_2, \ldots, \alpha_N \right\},$$
$$R_k = \mathbb{E}\left\{ \nu_k \nu_k^T \right\} = \text{diag}\left\{ R_k^1, R_k^2, \ldots, R_k^N \right\}.$$

Now we are in a position to present the aim of this paper. In view of the observation sequence $\{y_1, y_2, \ldots, y_k\}$, we will design the linear optimal filter in the sense of minimum variance for the addressed stochastic systems (1)–(2) with state delay and missing measurements, and derive new optimal filtering algorithm.

3. Main results

Now, the following definitions are introduced in order to facilitate the subsequent derivations.

Definition 1. Set $\tilde{x}_{i|k} = x_i - \hat{x}_{i|k}$. Then, define $\Phi_{k(i,j)} = \mathbb{E}\left\{ \tilde{x}_{i|k} \tilde{x}_{j|k}^T \right\}$, where $i \neq j$. Particularly, $P_{i|k} = \Phi_{k(i,i)} = \mathbb{E}\left\{ \tilde{x}_{i|k} \tilde{x}_{i|k}^T \right\}$, if $i = j$. Moreover, $\Phi_{k(i,j)} = \Phi_{k(j,i)}^T$.

Definition 2. Let $\Theta_{(k,j)} = \mathbb{E}\left\{ x_k x_j^T \right\}$, for $k \neq j$. Particularly, $\Theta_{(k,k)} = \mathbb{E}\left\{ x_k x_k^T \right\}$, if $k = j$. In addition, $\Theta_{(k,j)} = \Theta_{(j,k)}^T$. Then, we can obtain the recursion of $\Theta_{(k+1,k+1)}$ as

$$\Theta_{(k+1,k+1)} = A_k \Theta_{(k,k)} A_k^T + A_k \Theta_{(k,k-d)} A_{k-d}^T + A_{k-d} \Theta_{(k-d,k-d)} A_{k-d}^T + B_k Q_k B_k^T. \tag{5}$$

Definition 3. Define $\Sigma_k^t = \Theta_{(k-t,k-d)}$ for $t = 0, 1, \ldots, d$. Then, one has the recursion of $\Sigma_k{}^t$ given by: