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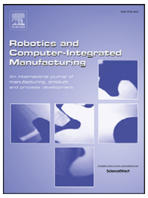

A method for stiffness analysis of overconstrained parallel robotic mechanisms with Scara motion

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ABSTRACT

This paper presents a general method for analyzing stiffness of overconstrained parallel robotic mechanisms with Scara motion. In the method, the stiffness model of a limb is derived by applying Castigliano's second theorem to strain energy of the limb with a structural decomposition strategy, and the stiffness matrix of a parallel mechanism is established based on stiffness models of limbs and the static equilibrium equation of the moving platform. Comparisons show that the stiffness model obtained from the proposed method is very close to the counterpart obtained from finite element analysis (FEA). In addition, a new index is proposed to evaluate stiffness performance of a parallel mechanism in a given configuration based on strain energy under external unit forces and moments. With this index, the dimensions of a parallel mechanism can be optimized and the path of a given task can be planned to obtain high stiffness.

1. Introduction

The end-effector of the well-known SCARA serial robot outputs three-translation and one-rotation (3T1R) motion (also called Scara motion or Schönflies motion). As parallel counterparts of the SCARA robot, 3T1R parallel manipulators attracted great attention of many scholars, and plenty of overconstrained and non-overconstrained 3T1R parallel manipulators have been invented [1–4]. Stiffness (or compliance) is one of the most important performances for many parallel robotic manipulators [5], such as machine tools and surgical robot. Stiffness modeling of some 3T1R non-overconstrained parallel mechanisms has been investigated, such as the H4 mechanism [6], and the 2PUS–2PRS mechanism [7], in which P, R, U and S denote a prismatic pair, revolute pair, universal pair and spherical pair, respectively. However, to our best knowledge, little literature reported stiffness analysis of 3T1R overconstrained mechanisms, whose stiffness modeling is a complicated problem.

The existing methods for stiffness modeling of parallel mechanisms can be divided into four categories: the FEA method, the structural matrix method, the virtual work principle method and the strain energy method. Generally, the FEA method is used to identify stiffness coefficients of links or joints, and validate computational results of analytical methods or experiment results [8]. Considering computational complexity, the structural matrix method is little used for stiffness modeling of parallel mechanisms [9].

The virtual work principle method has been widely used to derive stiffness models of overconstrained parallel mechanisms and non-overconstrained ones. Gosselin [10] first derived a stiffness model of the Stewart parallel mechanism with only considering compliance of actuators. Huang et al. [8] investigated stiffness estimation of a tripod-based parallel machine with considering compliance of spherical joints and lead-screw assemblies. Dai and Ding [11] analyzed compliance of a three-legged rigidly-connected compliant platform device. Majou et al. [12] dealt with parametric stiffness analysis of the Orthoglide mechanism with flexible links and joints. Xu and Li [13] derived the stiffness model of a translational parallel manipulator with compliant actuators and links. Chen et al. [14] derived the stiffness model of 3SPS+1PS bionic parallel test platform. By dividing the stiffness model of a limb into two parts along its actuation wrench and constraint wrench, stiffness analysis of some parallel manipulators has been investigated [15]. Sun et al. [16–18] systematically analyzed the stiffness performances of some novel and promising parallel robots considering linear and angular stiffness couplings of limbs. A called virtual joint method was first proposed by Zhang and Gosselin [19–21] for stiffness analysis of parallel mechanisms. Pashkevich et al. [9,22] presented systematically a modified virtual joint method for stiffness modeling of overconstrained parallel manipulators with flexible links and joints.

Recently, the strain energy method has been developed for stiffness modeling of some non-overconstrained parallel mechanisms. By applying Castigliano's second theorem to strain energy of the whole mechanism, Rezaei et al. [23] derived the stiffness model of the 3-PSP parallel mechanism, and Yan et al. [24] established the stiffness model of the Delta parallel robotic mechanism.

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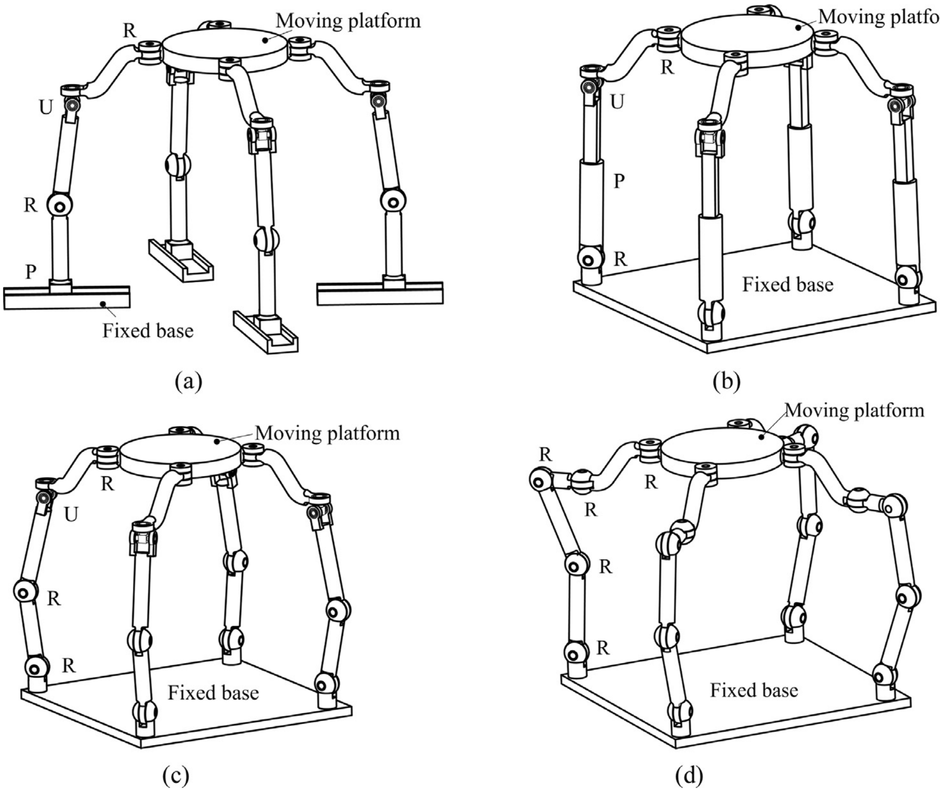

Fig. 1. Typical overconstrained parallel mechanisms with Scara motion: (a) 4-PRUR mechanism, (c) 4-RPUR mechanism, (d) 4-RRUR mechanism, (e) 4-RRRRR mechanism.

Comparing with the virtual work principle method, the strain energy method has lower computational efforts since coordinate transformations of deformations of links and joints are not necessary to consider. It is appointed out that the first step of the existing strain energy method for stiffness modeling is to solve applied wrenches exerted on links and joints under external wrenches [23,24]. However, applied wrench analysis of overconstrained parallel mechanisms involves the statically indeterminate problem, whose solution requires first stiffness models. Up to now, the contradiction between stiffness modeling and applied wrench analysis of overconstrained parallel mechanisms has not been dealt with well in the traditional strain energy method.

In our previous work [25,26], stiffness modeling of 3R2T and 3T2R overconstrained parallel mechanisms has been investigated based on a new method, which is different from the virtual work principle method and the traditional strain energy method. In this paper, the method will be extended to overconstrained parallel mechanisms with Scara motion. What is more, a new index is proposed to evaluate stiffness performance of a parallel mechanism in a given configuration.

The remainder of this paper is organized as follows. Expressions of joint applied wrench are derived in Section 2. Stiffness modeling of limbs is presented in Section 3. The stiffness modeling of mechanisms is shown in Section 4. Compare the results from the proposed method with the ones from a FEA model in Section 5. Section 6 presents a new index to evaluate the stiffness performance of a parallel mechanism in a given configuration. Finally, some conclusions of the work are drawn in Section 7.

2. Expressions of joint applied wrenches

2.1. Inverse displacement analysis

Fig. 1 shows some typical overconstrained parallel mechanisms with Scara motion, such as 4-PRUR, 4-RPUR, 4-RRUR and 4-RRRRR, in which each limb can be considered to be composed of a 2 degree-of-freedom (DOF) planar subchain and a 3-DOF planar subchain [3,27], and all R pairs connected to the moving platform are perpendicular to the platform plane.

To derive expressions of joint applied wrenches, the direction vector and position vector of each pair with respect to a fixed reference frame need to be determined through inverse displacement analysis.

A fixed coordinate frame *A: XYZ* can be attached to a point *O* in the fixed base, with *Z* axis perpendicular to the base plane, and *X* axis and *Y* axis in the plane. A moving coordinate frame *B: xyz* can be attached to central point p_0 of the moving platform, with *z* axis perpendicular to the platform plane, and *x* axis and *y* axis in the plane. Based on mobility analysis [3,4], the moving platform of a typical 3T1R mechanism cannot rotate about *X* axis and *Y* axis of frame *A*. So, the orientation matrix ${}^{A}_{B}\boldsymbol{R}$of the moving platform is

$$
{}^{A}_{B}\boldsymbol{R} = \boldsymbol{R}(z,\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}
$$

where α denotes rotational angle of the frame *B* about *Z* axis of the fixed frame *A*. The position of the moving platform is represented as

$$
\boldsymbol{r}_{p_0} = \begin{pmatrix} p_x & p_y & p_z \end{pmatrix}^T \tag{2}
$$

where p_x, p_y and p_z denote three position coordinates of point p_0.

Some equations can be set up easily based on the lengths of links and the relations between joint axes. The position vector and direction vector of each pair can be expressed as the function of the independent position-orientation parameters p_x, p_y, p_z and α, based on those equations.

2.2. Expressions of joint applied wrenches

Based on the reciprocal relation between constraints and twists in screw theory [28,29], the unit constraint wrench of a limb exerted on the moving platform can be calculated according to its twist system,