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Order restricted inference of a multiple step-stress model

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ABSTRACT

In this manuscript both the classical and Bayesian analyses of a multiple step-stress model have been considered. The lifetime distributions of the experimental units at each stress level follow two-parameter generalized exponential distribution and they are related through the cumulative exposure model assumptions. Recently Abdel-Hamid and AL-Hussaini (2009) provided the classical inference of the model parameters of a simple step-stress model, under the same set of assumptions. In a typical step-stress experiment, it is expected that the lifetime of the experimental units will be shorter at the higher stress level. The main aim of this paper is to develop the order restricted inference of the model parameters of a multiple step-stress model based on both the classical and Bayesian approaches. An extensive simulation study has been performed and one data set has been analyzed for illustrative purposes.

1. Introduction

In today's competitive world the industrial products become highly reliable. Therefore, for any analysis purposes, it becomes very difficult to get sufficient failure time data during the normal experimental time. The accelerated life testing (ALT) experiment is frequently being used to overcome this problem. The ALT experiments are introduced to conduct the experiment under one or more extreme operating conditions and thus increasing the number of failures within an affordable experimental time. The factors which directly affect the lifetime of the products are called stress factors, for example, voltage, temperature, humidity could be some of the stress factors for testing an electronic equipment. Some of the key references on different ALT models are Nelson (1980), Bagdonavicius and Nikulin (2002) and the references cited therein.

A special case of the ALT experiment is known as the step-stress life testing (SSLT) experiment, where the stress changes at a given time or after a specified number of failures. In a SSLT experiment if we consider only two stress levels, then it is known as a simple step-stress experiment. In a review article Balakrishnan (2009) extensively discussed different inferential issues of a step-stress model when the lifetime distributions of the experimental units follow exponential distribution. In a recent monograph, Kundu and Ganguly (2017) provided an extensive review of the different step-stress models.

Let us assume that the cumulative distribution function (CDF) of the lifetime at the stress level S_{i-1} is $F_i(\cdot)$. To analyze a data obtained from a SSLT experiment, one needs a model which relates the CDFs of lifetime under different stress levels to the CDF of the lifetime of the product under the SSLT experiment. Several models are available in the literature to describe this relationship. The most popular one is known as the cumulative exposure model (CEM) originally proposed by Sedyakin (1966) and later quite extensively studied by Bagdonavicius (1978) and Nelson (1980). This model assumes that the remaining lifetime of an experimental unit depends only on the cumulative exposure accumulated at the current stress level, irrespective of how the exposure has actually been accumulated. An extensive amount of work has been done

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in developing the statistical inference of the model parameters in a SSLT set up under the CEM assumptions, see for example the recent Ph.D. thesis by Ganguly (2013) or the monograph by Kundu and Ganguly (2017) for an extensive list of references on different step-stress models.

The main objective of a SSLT experiment is to reduce the lifetime of the experimental units by increasing the stress level. Therefore, it is quite natural to assume that the expected lifetime of the experimental units is lower at the higher stress level. Balakrishnan et al. (2009) first incorporated this information and considered the estimation of the model parameters based on the assumption that the lifetime distribution of the experimental units follow exponential distribution. They obtained the order restricted maximum likelihood estimators (MLEs) and also discussed the hypothesis testing problems under order restrictions in case of Type-I and Type-II censored data. Recently, Samanta et al. (2017) developed the order restricted Bayesian inference of the model parameters under the same set of assumptions.

A two-parameter generalized exponential (GE) distribution has received a considerable amount of attention since its introduction by Gupta and Kundu (1999). A two-parameter GE distribution with the shape parameter $\alpha > 0$ and scale parameter $\theta > 0$ has the following CDF, probability density function (PDF) and hazard function (HF), respectively,

$$F(t;\alpha,\theta) = (1-e^{-\theta t})^{\alpha}, \qquad t > 0, \tag{1}$$

$$f(t;\alpha,\theta) = \theta\alpha e^{-\theta t}(1-e^{-\theta t})^{\alpha-1}, \qquad t > 0, \tag{2}$$

$$H(t;\alpha,\theta) = \frac{\alpha\theta(1-e^{-\theta t})^{\alpha-1}e^{-\theta t}}{1-(1-e^{-\theta t})^{\alpha}}, \qquad t > 0. \tag{3}$$

From now on a GE distribution with the shape parameter α and the scale parameter θ will be denoted by GE(α, θ). Due to presence of the shape parameter the GE distribution is a very flexible model. The PDF of a GE distribution can be a decreasing or a unimodal function. Moreover, the hazard function of a GE distribution can be an increasing, decreasing or a constant function depending on the shape parameter. Conventional exponential distribution is a special case of the GE distribution. Therefore, Weibull, gamma and the GE distributions are all extensions of the exponential distribution but in different ways. It has been shown by Gupta and Kundu (2001) that the GE distribution can be a good alternative to a gamma or a Weibull distribution. In fact in many cases it may provide a better fit to a given data set, than the gamma or the Weibull distribution. Interested readers are referred to Gupta and Kundu (2007), Al-Hussaini and Ahsnullah (2015), Nadarajah (2011) and the references cited therein for different developments associated with the GE distribution.

In this paper we consider the analysis of a given data set, obtained from a multiple SSLT experiment. It is assumed that the lifetime distribution of the experimental unit under each stress level follows a two parameter GE distribution with the same shape parameter but different scale parameters, and it satisfies the CEM assumptions. It is further assumed that the expected lifetime of the experimental units at the higher stress level is smaller compared to a lower stress level. We provide the order restricted inference of the model parameters both under the classical and Bayesian set up. We provide both the point and interval estimators of the unknown parameters associated with the model. An extensive simulation experiment has been performed to see the effectiveness of the order restricted inference under both classical and Bayesian methods. It is observed that the performances of the Bayes estimators even with non-informative priors are significantly better than the classical estimators in terms of biases and mean squared errors (MSEs). We provide the analysis of one data set for illustrative purposes.

The rest of the paper is organized as follows. In Section 2, we provide the model assumptions and the likelihood function based on the available data. In Section 3, we obtain the MLEs and the associated Fisher information matrices of the unknown parameters. The Bayes estimators and their credible intervals are provided in Section 4. In Section 5 we present the simulation results and the analysis of one data set. Finally we conclude the paper in Section 6.

2. Model assumptions and the likelihood function

Let us assume that there are $m+1$ stress levels, say $S_0, S_1, \ldots, S_m$ and the expected lifetime of experimental units is shorter at the stress level S_k than at the stress level S_{k-1}; $k = 1, \ldots, m$. Suppose n experimental units are subjected to a life testing experiment at the time point 0, under the stress level S_0. The stress level is increased to S_1 at a pre-fixed time τ_1 and then to S_2 at a pre-fixed time τ_2 and so on. Finally the stress level is increased to S_m at the time point τ_m, and the experiment continues till all the n items fail.

Failure time data obtained from this multiple SSLT experiment is denoted by

$$\mathcal{D} = \{t_{1:n} < \cdots < t_{n_1:n} < \tau_1 < t_{n_1+1:n} < \cdots < t_{n_1+n_2:n} < \tau_2 < \cdots < \tau_m < t_{(n_1+\cdots+n_m+1):n} < \cdots < t_{n:n}\}. \tag{4}$$

Here n_k is the number of failures under the stress level S_{k-1} $(k = 1, \ldots, m+1)$. It is assumed that the lifetime distribution of the experimental units under the stress level S_{k-1} follows GE(α, θ_k). Hence, for $\alpha > 0, \theta_k > 0$ and $t > 0$,

$$F_k(t) = \left(1-e^{-\theta_k t}\right)^{\alpha}, \quad k = 1, \ldots, m+1.$$

Since it is assumed that $F_1(\cdot), \ldots, F_{m+1}(\cdot)$ satisfy the CEM assumptions, we have

$$F(t) = \begin{cases} F_1(t) & \text{if } 0 < t \le \tau_1, \\ F_k(c_{k-1}+t-\tau_{k-1}) & \text{if } \tau_{k-1} < t < \tau_k, \\ F_{m+1}(c_m+t-\tau_m) & \text{if } \tau_m < t < \infty, \end{cases} \tag{5}$$