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Solving the train marshalling problem by inclusion–exclusion

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ABSTRACT

In the Train Marshalling Problem (TMP) the cars of a train having different destinations have to be reordered in such a way that all the cars with the same destination appear consecutively. To this aim the cars are first shunted on k auxiliary rails, then the sequences of cars present on the different rails are reconnected one after the other to form a new train. The TMP is the problem of minimizing the number k of auxiliary rails needed to obtain a train with the required property. The TMP is an NP-hard problem. Here we present an exact dynamic programming algorithm for the TMP based on the inclusion–exclusion principle. The algorithm has polynomial space complexity and time complexity that is polynomial in the number of cars, exponential in the number of destinations. This shows that the TMP is fixed parameter tractable with the number of destinations as parameter.

1. Introduction

In the Train Marshalling Problem (TMP) a train $T = (a_1, a_2, \ldots, a_n)$ with n cars having t different destinations is given. The order of the cars of the train can be modified by means of k auxiliary rails (or tracks) as follows. First the n cars $a_1, a_2, \ldots, a_n$ are considered one by one in their initial order and each car is moved to one of the k auxiliary rails and placed behind the cars already on the rail. This process creates k sequences of cars, one for each rail. Then a new train is created by reconnecting these sequences in a single one by sequencing the cars on the first rail in their actual order followed by the cars on the second rail and so on, ending with the cars on the kth rail. The TMP is the problem of finding the minimum number k of auxiliary rails needed to rearrange the train in such a way that all the cars with the same destination appear consecutively. An order of the cars that satisfies this condition will be called a TM-order. The decision version of the problem (DTMP) is the problem of deciding if, given a natural number k, a TM-order of a train T can be obtained by using at most k auxiliary rails.

The TMP was originally proposed in [15] by Zhu and Zhu who studied some polynomial classes of the problem. In [7] Dahlhaus et al. showed that the DTMP is NP-complete and, as a consequence, the TMP is an NP-hard problem. They also proved that the optimal value of the TMP is upperbounded by the value

$$L(n, t) = \min\{t, \lceil n/4 + 1/2 \rceil\}. \quad (1)$$

A 2-approximation algorithm for the TMP based on an interval graph coloring approach has been proposed by Dahlhaus et al. in [8]. The question if the TMP is a fixed parameter tractable problem has been addressed by Brueggeman et al. in [6]. In this paper the authors describe a dynamic programming procedure for the DTMP that requires time $O(2^{O(k)} poly(n))$ and

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space $O(n^2k2^{8k})$, thus polynomial for each fixed value of the parameter k. However, the proof of correctness of the algorithm contains a bug which, to the authors' knowledge, has not been fixed yet [9].

In this paper we present an exact algorithm that solves a graph theoretical model of the DTMP. The algorithm is a dynamic programming procedure based on the principle of inclusion–exclusion and has time complexity $O(nkt^2 2^t)$ and polynomial space complexity $O(nkt)$. This procedure can be easily adopted to solve the TMP by binary search in $O(nt^2 2^t L \log_2 L)$ where $L = L(n, t)$ is the upperbound given in (1). In particular, this shows that the TMP is fixed parameter tractable with respect the number t of different destinations. Because of the exponential factor 2^t in the time complexity, the procedure can be reasonably used to solve instances with a number of destinations t up to 30. It remains an open problem if the DTMP shows the stronger property to be fixed parameter tractable even with respect to the number k of auxiliary rails.

The TMP belongs to a wide class of combinatorial problems which arise in the optimization of the train classification processes. These problems usually require to rearrange the cars of a train (or of a set of trains) to sequence them in an assigned order. So, differently from the TMP, the final order of the cars is usually an input of the problem. The goal is reached by partitioning the cars of the train on a given set of auxiliary tracks and then performing a sequence of so called *roll-in* operations, where each roll-in operation takes the cars on a track and suitably routes them on the other tracks. The main objective is either the minimization of the number of roll-in operations or the number of cars globally involved in these operations. Furthermore, scheduling aspects can also affect the problem. For a detailed description of the train classification problems see for instance [10,11] and the references therein.

We remind that the inclusion–exclusion principle has been successfully applied in combinatorial optimization to solve classical NP-problems, in particular graph theory problems. Initially, Karp [12] used this approach to solve the hamiltonian path problem, the bin packing problem and some sequencing problems. Later on, other combinatorial problems have been addressed by the inclusion–exclusion approach in [1–4]. In particular, Björklund et al. [5] have proposed a general way to solve a class of set partitioning problems including chromatic number, dominating number, maximum k-cut and list coloring.

The remainder of the paper is organized as follows. In Section 2 we formally define the TMP. In Section 3 we introduce a graph theoretical model for the DTMP suitable to be solved by the inclusion–exclusion principle. In Section 4 we present two dynamic programming procedures that solve the DTMP and the TMP, respectively. Some conclusions are drawn in Section 5.

Notation. We will use the following notation. For each $n \in \mathbb{N}$, $\mathbb{N}_n = \{1, \ldots, n\}$ denotes the set of integers in between 1 and n and for $i, j \in \mathbb{N}$, $i < j$, we denote by $[i, j]$ the set of integers r with $i \le r \le j$. Given a sequence α of length n and a subset $S \subset \mathbb{N}_n$ we denote by $\alpha[S]$ the sequence obtained from α by removing the elements in positions in $\mathbb{N}_n \setminus S$. If $S = \{i\}$, we also write $\alpha[S] = \alpha[i] = \alpha_i$. Given two sequences α and β of length m and n, respectively, we denote by $\alpha \cdot \beta$ the concatenation of α and β, that is the sequence $(\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_n)$. Finally, any (total) order of a set $\mathbb{N}_n$ is represented by the sequence τ of length n where $\tau[i]$, $i = 1, \ldots, n$, denotes the ith element in the order.

2. Problem formulation

An instance of the TMP consists in a triple $(n, t, \mathcal{D})$ where n is the number of cars of the train, t is the number of destinations and $\mathcal{D}$ is a partition of $\mathbb{N}_n$ in subsets $D(j), j \in \mathbb{N}_t$. Each set $D(j)$ contains the indices of the cars having destination j. For the sake of simplicity, in the following we identify the cars of the train with their index. In this way the original order of the cars corresponds to the sequence $(1, 2, \ldots, n)$.

Definition 1. An order τ of $\mathbb{N}_n$ is said a TM-order for the instance $(n, t, \mathcal{D})$ if the elements of each set $D(j), j \in \mathbb{N}_t$, appear consecutively in τ, i.e., $\tau[r], \tau[s] \in D(j)$ for some $1 \le r < s \le n$ implies $\tau[i] \in D(j)$ for every $r \le i \le s$.

To formally define the TMP we observe that the train $\alpha = (1, 2, \ldots, n)$ can be reordered by means of k auxiliary tracks to obtain a TM-train if and only if there exists a map $\phi : \mathbb{N}_n \to \mathbb{N}_k$ such that, setting $\phi^{-1}(r) = \{i \in \mathbb{N}_n : \phi(i) = r\}$ for each $r \in \mathbb{N}_k$, the sequence

$$\tau^\phi = \alpha[\phi^{-1}(1)] \cdot \alpha[\phi^{-1}(2)] \cdots \alpha[\phi^{-1}(k)]$$

is a TM-order of $\mathbb{N}_n$. In this case the map ϕ is said a k-TM-solution, or briefly a k-solution, of the TMP instance. Each set $\phi^{-1}(r)$ denotes the cars that are moved on the rth track. According to $\alpha[\phi^{-1}(r)]$, these cars are sequenced on track r for increasing indices. Let us denote by π^ϕ the order of N_t in which the t destinations appear in τ^ϕ, i.e., such that for each $j \in \mathbb{N}_t$ the cars in $D(\pi^\phi(j))$ appear in τ^ϕ just after the cars in $D(\pi^\phi(j-1))$.

The TMP and its decision version DTMP can be formulated as follows.

Train Marshalling Problem: Given a TMP instance $(n, t, \mathcal{D})$ find the minimum $k \in \mathbb{N}$ such that there exists a k-solution.

Decision Train Marshalling Problem (DTMP). Given a TMP instance $(n, t, \mathcal{D})$ and $k \in \mathbb{N}$, determine if there exists a k-solution.

Example 1. Consider the TMP instance defined by $n = 10$, $t = 5$ and

$$D_1 = \{1, 8\}, \quad D_2 = \{2, 9\}, \quad D_3 = \{3, 7\}, \quad D_4 = \{4, 6\}, \quad D_5 = \{5, 10\}.$$