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Maximum weight independent sets in classes related to claw-free graphs

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ABSTRACT

The MAXIMUM WEIGHT INDEPENDENT SET (MWIS) problem on graphs with vertex weights asks for a set of pairwise nonadjacent vertices of maximum total weight. The complexity of the MWIS problem for $S_{1,1,3}$-free graphs, and for $S_{1,2,2}$-free graphs is unknown. We show that the MWIS problem in ($S_{1,1,3}$, banner)-free graphs, and in ($S_{1,2,2}$, bull)-free graphs can be solved in polynomial time. These results extend some known results in the literature.

1. Introduction

In an undirected graph G, an *independent set* is a subset of mutually nonadjacent vertices in G. The MAXIMUM INDEPENDENT SET (MIS) problem asks for an independent set of G with maximum cardinality. The MAXIMUM WEIGHT INDEPENDENT SET (MWIS) problem asks for an independent set of total maximum weight in the given graph G with vertex weight function w on $V(G)$. The M(W)IS problem ([GT20] in [13]) is one of the fundamental algorithmic graph problems which frequently occurs as a subproblem in models in computer science, bioinformatics, operations research and other fields. Also, the problem has numerous applications, including train dispatching [12] and data mining [26]. The MWIS problem is known to be *NP*-complete in general and hard to approximate; it remains *NP*-complete even on restricted classes of graphs such as triangle-free graphs [24], and ($K_{1,4}$,diamond)-free graphs [9]. Alekseev [1] showed that the M(W)IS problem remains *NP*-complete on H-free graphs, whenever H is connected, but neither a path nor a subdivision of the claw. On the other hand, the M(W)IS problem is known to be solvable in polynomial time on many graph classes by various techniques; see [1,2,4,6,7,14,17,20,21].

Here we will focus on graphs without a subdivision of a claw. For integers $i, j, k \geq 1$, let $S_{i,j,k}$ denote a tree with exactly three vertices of degree one, being at distance i, j and k from the unique vertex of degree three. The graph $S_{1,1,1}$ is called a *claw* and $S_{1,1,2}$ is called a *chair or fork*. Also, note that $S_{i,j,k}$ is a subdivision of a claw. Let $K_{p,q}$ denote the complete bipartite graph with p vertices in one partition set and q vertices in the other. Let P_n denote the path on n vertices.

Minty [22] reduced the MWIS problem on claw-free graphs to the Maximum Matching problem, and thus gave a polynomial time algorithm for the MWIS problem on claw-free graphs. Recently, the MWIS problem in extensions of claw-free graphs have received much attention. Using modular decomposition techniques, Lozin and Milanič [20] showed that the MWIS problem can be solved in polynomial time for the class of fork-free graphs, and using a combination of modular and clique separator decomposition techniques, Brandstädt et al. [6] proved that the MWIS problem can be solved in polynomial time for the class of apple-free graphs. It is also known that the MIS problem can be solved in polynomial time for some

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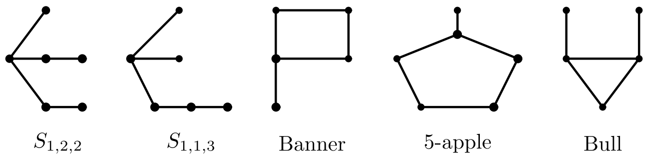

Fig. 1. Some special graphs.

subclasses of $S_{i,j,k}$-free graphs; see [14, Table 1]. However, the complexity of the MWIS problem is unknown for the class of $S_{1,1,3}$-free graphs, and for the class of $S_{1,2,2}$-free graphs. In particular, the class of P_6-free graphs, the class of $S_{1,2,2}$-free graphs, and the class of $S_{1,1,3}$-free graphs constitute the minimal classes, defined by forbidding a single connected subgraph on six vertices, for which the computational complexity of M(W)IS problem is unknown.

Note that the class of $S_{1,1,3}$-free graphs and the class of $S_{1,2,2}$-free graphs extend the class of fork-free graphs and the class of P_5-free graphs. It is also known that the MWIS problem in P_5-free graphs can be solved in polynomial time [19].

The *bull* is the graph with five vertices a, b, c, d, e and edges ab, bc, cd, be, ce. The *house* is the graph with five vertices a, b, c, d, e and edges ab, bc, cd, de, ae, be (i.e., the complementary graph of P_5). The *k-apple* is the graph obtained from a chordless cycle C_k of length $k \geq 4$ by adding a vertex that has exactly one neighbor on the cycle. The 4-apple is also called a *banner*. See Fig. 1 for some of the special graphs used in this paper. If $\mathcal{F}$ is a family of graphs, a graph G is said to be $\mathcal{F}$*-free* if it contains no induced subgraph isomorphic to any graph in $\mathcal{F}$.

We follow the approach developed recently by Brandstädt and Giakoumakis [3], which combines modular decomposition and clique separator decomposition; and we show the following:

(i) The MWIS problem can be efficiently solved in the class of ($S_{1,1,3}$, banner)-free graphs. This result extends some known results in the literature such as: the aforementioned results for claw-free graphs and P_4-free graphs, (P_5, banner)-free graphs [18], and a result on the MIS problem in ($S_{1,1,3}$, banner)-free graphs [16].

(ii) The MWIS problem can be efficiently solved in the class of ($S_{1,2,2}$, bull)-free graphs. This result extends some known results in the literature such as: P_4-free graphs, (chair, bull)-free graphs [4], and (P_5, bull)-free graphs [7].

2. Notation, terminology, and preliminaries

For notation and terminology not defined here, we follow [5]. Let G be a finite, undirected and simple graph with vertex-set $V(G)$ and edge-set $E(G)$. We let $|V(G)| = n$ and $|E(G)| = m$. We let $\overline{G}$ denote the complementary graph of G. For a vertex $v \in V(G)$, the *neighborhood* $N(v)$ of v is the set $\{u \in V(G) \mid uv \in E(G)\}$, and its *closed neighborhood* $N[v]$ is the set $N(v) \cup \{v\}$. The neighborhood $N(X)$ of a subset $X \subseteq V(G)$ is the set $\{u \in V(G) \setminus X \mid u \text{ is adjacent to a vertex of } X\}$, and its closed neighborhood $N[X]$ is the set $N(X) \cup X$. Given a subgraph H of G and $v \in V(G) \setminus V(H)$, let $N_H(v)$ denote the set $N(v) \cap V(H)$, and for $X \subseteq V(G) \setminus V(H)$, let $N_H(X)$ denote the set $N(X) \cap V(H)$. For any two subsets $S, T \subseteq V(G)$, we say that S is *complete* to T if every vertex in S is adjacent to every vertex in T.

A vertex $z \in V(G)$ *distinguishes* two other vertices $x, y \in V(G)$ if z is adjacent to one of them and nonadjacent to the other. A set $M \subseteq V(G)$ is a *module* in G if no vertex from $V(G) \setminus M$ distinguishes two vertices from M. The *trivial modules* in G are $V(G)$, $\emptyset$, and all one-vertex sets. A graph G is *prime* if it contains only trivial modules. Note that prime graphs on at least three vertices are connected.

A class of graphs $\mathcal{G}$ is *hereditary* if every induced subgraph of a member of $\mathcal{G}$ is also in $\mathcal{G}$. We will use the following theorem by Lozin and Milanič [20].

Theorem 1 ([20]). *Let $\mathcal{G}$ be a hereditary class of graphs. If the MWIS problem can be solved in $O(n^p)$-time for prime graphs in $\mathcal{G}$, where $p \geq 1$ is a constant, then the MWIS problem can be solved for graphs in $\mathcal{G}$ in time $O(n^p + m)$.* □

A *clique* in G is a subset of pairwise adjacent vertices in G. A *clique separator* (or *clique cutset*) in a connected graph G is a subset Q of vertices in G which induces a complete graph, such that the graph induced by $V(G) \setminus Q$ is disconnected. A graph is an *atom* if it does not contain a clique separator.

Let $\mathcal{C}$ be a class of graphs. A graph G is *nearly* $\mathcal{C}$ if for every vertex v in $V(G)$ the graph induced by $V(G) \setminus N[v]$ is in $\mathcal{C}$. Let $\alpha_w(G)$ denote the weighted independence number of G. Obviously, we have:

$$\alpha_w(G) = \max\{w(v) + \alpha_w(G \setminus N[v]) \mid v \in V(G)\}. \tag{1}$$

Thus, whenever MWIS is solvable in time T on a class $\mathcal{C}$, then it is solvable on nearly $\mathcal{C}$ graphs in time $n \cdot T$. Based on Eq. (1), one can obtain the following theorem, whose proof is essentially the same as in [27, Section 3.4] and hence is omitted.

Theorem 2. *Let $\mathcal{C}$ be a class of graphs such that MWIS can be solved in time $O(f(n))$ for every graph in $\mathcal{C}$ with n vertices. Then in any hereditary class of graphs whose atoms are all nearly $\mathcal{C}$ the MWIS problem can be solved in time $O(n^2 \cdot f(n))$.* □

The following notation will be used several times in the proofs. Given a graph G, let v be a vertex in G and H be an induced subgraph of $G \setminus N[v]$. Let $t = |V(H)|$. Then we define the following sets: