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Kernelization of edge perfect code and its variants

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ABSTRACT

We show that the three problems Edge Perfect Code, Edge Weakly-Perfect Code, and Edge Semi-Perfect Code are all fixed-parameter tractable (with the solution size k as parameter) by obtaining various kernelization results. In general graphs, Edge Perfect Code admits a problem kernel with $O(k^2)$ vertices and $O(k^3)$ edges, Edge Weakly-Perfect Code admits a problem kernel with $O(k^3)$ vertices and $O(k^3)$ edges, and Edge Semi-Perfect Code admits a problem kernel with $O(2^k \cdot k)$ vertices and $O(2^k \cdot k)$ edges. In planar graphs and in graphs without small cycles or large stars, the kernel sizes for the three problems can be significantly reduced, to $O(k^2)$ and in some cases even to $O(k)$. On the other hand, all three problems remain NP-complete in grid graphs of maximum degree 3 and arbitrarily large girth. Moreover, Edge Semi-Perfect Code does not admit any polynomial kernel in bipartite graphs unless NP $\subseteq$ coNP/poly.

1. Introduction

Let $G = (V, E)$ be a simple undirected graph. Each edge $uv \in E$ *dominates* itself and all edges incident to either u or v. An *edge dominating set* is a subset of edges D in G such that every edge in E is dominated by at least one edge in D. An edge dominating set D is an *edge perfect code* if every edge in E is dominated by exactly one edge in D. An edge dominating set D is an *edge semi-perfect code* if every edge in $E \setminus D$ is dominated by exactly one edge in D (note that the edges in an edge semi-perfect code may be adjacent to each other). An edge semi-perfect code D is an *edge weakly-perfect code* if every edge in D is adjacent to at most one other edge in D.

For example, in the path P_5, which consists of four edges, the only edge perfect code must consist of the two outer edges, while both the two inner edges and the two outer edges can be an edge weakly-perfect code. For another example, in the graph obtained by subdividing each edge of the star $K_{1,3}$ into two edges, the only edge perfect code, and the only edge weakly-perfect code, is the set of three outer edges incident to the three leaves, while an edge semi-perfect code may be composed of the three inner edges incident the degree-three vertex, or even of all six edges of the graph.

Definition 1. Given a graph G and a parameter k, Edge Perfect Code (respectively, Edge Weakly-Perfect Code, Edge Semi-Perfect Code) is the problem of deciding whether G has an edge perfect code (respectively, edge weakly-perfect code, edge semi-perfect code) of at most k edges.

The problem Edge Perfect Code has applications in parallel resource allocation, coding theory, and network routing, and is NP-complete even in planar bipartite graphs of maximum degree 3 [15,23,22,4]. On the other hand, it is polynomial-time solvable in series–parallel graphs [15], bipartite permutation graphs [23], chordal graphs [22], hole-free graphs [4],

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claw-free graphs [6], dually chordal graphs [5], and biconvex graphs [21]. Very recently, exponential-time exact algorithms [20,29] and a polynomial kernel [28] were proposed for Edge Perfect Code in general graphs.

The name of "perfect code" originates from the graph formulation of a related problem in coding theory [2]. Edge Perfect Code has also been studied under the names Efficient Edge Domination and Dominating Induced Matching in the literature. A matching in a graph is *dominating* if its edges dominate all other edges of the graph. An *induced matching* is a matching in which no two vertices from two different edges are adjacent. Clearly, all maximal matchings are dominating matchings, but not all maximal matchings are induced matchings. Without the dominating requirement, the problem of deciding whether a graph has an induced matching of at least k edges is W[1]-hard in bipartite graphs, and admits a linear kernel in planar graphs [25]. Little is known about Edge Weakly-Perfect Code and Edge Semi-Perfect Code, except that the latter is NP-complete in bipartite graphs [22].

The vertex variants of all three problems, in which the dominating and dominated graph elements are vertices instead of edges, have also been studied in the literature. In fact, Edge Perfect Code, Edge Weakly-Perfect Code, and Edge Semi-Perfect Code in any graph G are simply Perfect Code, Weakly-Perfect Code, and Semi-Perfect Code, respectively, in the line graph $L(G)$. Perfect Code is a well-known and extensively studied NP-complete problem; see [24] for an overview of its computational complexity in various graph classes. Weakly-Perfect Code and Semi-Perfect Code are NP-complete in planar graphs [13]. In terms of parameterized complexity, Perfect Code is W[1]-complete in general graphs [11,7], but becomes fixed-parameter tractable when restricted to planar graphs [17] and effectively nowhere-dense graphs [8].

Edge Perfect Code, Edge Weakly-Perfect Code, and Edge Semi-Perfect Code are all restricted variants of Edge Dominating Set, the problem of deciding whether a graph G has an edge dominating set of at most k edges. Edge Dominating Set is known to admit a problem kernel with $O(k^2)$ vertices and $O(k^3)$ edges in general graphs [27], and a problem kernel with $O(k)$ vertices and edges in planar graphs [16,26]. Also, a general result of Xiao and Nagamochi [28, Theorem 3] on regular bipartition with $a = 0$ and $b = 1$ implies that Edge Perfect Code admits a problem kernel with $O(k^3)$ vertices. Extending these results, we show that the three problems Edge Perfect Code, Edge Weakly-Perfect Code, and Edge Semi-Perfect Code are all fixed-parameter tractable by proving various kernel bounds. Our contributions are the following:

- In Section 2, we show that in general graphs, Edge Perfect Code admits a problem kernel with $O(k^2)$ vertices and $O(k^3)$ edges, Edge Weakly-Perfect Code admits a problem kernel with $O(k^3)$ vertices and $O(k^3)$ edges, and Edge Semi-Perfect Code admits a problem kernel with $O(2^k \cdot k)$ vertices and $O(2^k \cdot k)$ edges. We also show that Edge Semi-Perfect Code does not admit any polynomial kernel even in bipartite graphs, unless NP $\subseteq$ coNP/poly.
- In Section 3, we show that the kernel sizes for the three problems can be significantly reduced in planar graphs and in graphs without small cycles or large stars. On the other hand, all three problems remain NP-complete in grid graphs of maximum degree 3 and arbitrarily large girth.

Preliminaries. Parameterized complexity is a common approach to dealing with NP-hard problems [12,16]. A *parameterized problem* is a language of the form (x, k), where x is the input instance, and k is a nonnegative integer called the *parameter*. A parameterized problem is *fixed-parameter tractable* if it can be solved in time $f(k) \cdot |x|^{O(1)}$, where f is a computable function solely dependent on k, and $|x|$ is the size of the input instance. On the other hand, many problems that are not known to be fixed-parameter tractable can be classified in an infinite hierarchy of complexity classes W[1] $\subseteq$ W[2] $\subseteq \cdots$. For example, with the solution size as parameter, Vertex Cover is fixed-parameter tractable, Clique and Independent Set are W[1]-complete, and Dominating Set is W[2]-complete.

An instance (x, k) of a parameterized problem may be reduced to a *problem kernel*, that is, transformed by a polynomial-time algorithm to an equivalent reduced instance (x', k') with $\max\{|x'|, k'\} \le g(k)$ for some function g solely dependent on k. We call such a transformation the *kernelization* process, and call the function g the *size* of the kernel. When g is a polynomial function, we say that the problem has a *polynomial kernel*. It is well-known that a decidable parameterized problem is fixed-parameter tractable if and only if it is kernelizable.

Let P and Q be parameterized problems. We say that P is *polynomial parameter reducible* to Q if there exists a polynomial time computable function $f : \{0, 1\}^* \times \mathbb{N} \to \{0, 1\}^* \times \mathbb{N}$ and a polynomial $p : \mathbb{N} \to \mathbb{N}$, such that for all $x \in \{0, 1\}^*$ and $k \in \mathbb{N}$, $(x, k) \in P$ if and only if $(x', k') = f(x, k) \in Q$, and $k' \le p(k)$. The function f is called a *polynomial parameter transformation* from P to Q. Let P^c and Q^c be the derived classical problems of P and Q, respectively, with the parameters encoded in unary. Suppose that P^c is NP-hard, Q^c is in NP, and there is a polynomial parameter transformation f from P to Q. Then P has polynomial kernel if Q has a polynomial kernel [3, Theorem 8].[1]

2. Kernelization in general graphs

We show that Edge Perfect Code, Edge Weakly-Perfect Code, and Edge Semi-Perfect Code are fixed-parameter tractable in general graphs by obtaining problem kernels for these problems. Let k-Edge Max-Degree-d Semi-Perfect

[1] In brief, since P^c is NP-hard and Q^c is in NP, there exists a polynomial-time transformation h from Q^c to P^c. Suppose that Q has a polynomial kernel, that is, there exists a polynomial-time algorithm g that transforms any instance (x, k) of Q to an equivalent reduced instance (x', k') with $|x'| + k' \le q(k)$ for some polynomial q. Then by composing f, g, and h, we can obtain a polynomial-time algorithm that transforms any instance (x, k), first by f to (x', k') with $k' \le p(k)$, second by g to (x'', k'') with $|x''| + k'' \le q(k') \le q(p(k))$, and third by h to (x''', k''') with $|x'''| + k'''$ polynomially bounded by $|x''| + k''$ and hence by k, such that $(x, k) \in P$ if and only if $(x', k') \in Q$ if and only if $(x'', k'') \in Q$ if and only if $(x''', k''') \in P$.