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Domain Theory its Ramifications and Interactions

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Abstract

At a conference that is devoted to a specific research area – Domain Theory in this case – there should be room for reflecting its rôle within the concert of scientific fields that sometimes cooperate and sometimes compete for recognition. It is the intention of this paper to stimulate interactions of Domain Theory with other research areas. Thus, this is not a research paper. It is the intention, firstly, to recall that Domain Theory has its historical roots in quite different fields, secondly, to indicate its interactions with other areas and not only with Computer Science, and thirdly. to put forward a new development in the theory of C*-algebras that opens new perspectives.

Keywords: Domain Theory and its history, Interactions with other areas, Cuntz semigroup, C*-algebra

1 Interaction versus Application

Competing for positions and grants, mathematical disciplines have to justify themselves. From the outside the impact on APPLICATIONS is often put forward as the relevant criterion. For this reason, titles of Journals, of Conferences, etc., like to take the form

[Something] and its Applications.

Whenever you see such a title, you can be sure that this is a Journal or a Conference on Pure Mathematics.

What is meant by 'applications' is quite subtle. Are these applications in fields outside of mathematics, like Engineering, Medicine, Computer Science (Physics)? Or applications in Statistics, Optimization, Numerical Methods? Or applications to other mathematical theories, or as background theories to applied mathematics?

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In this paper we put forward another criterion for judging the liveliness of a field of mathematical research: RAMIFICATIONS INTO and INTERACTIONS WITH other fields inside or outside of mathematics, no matter whether they are theoretical or applied.

Here, my question is: What are the ramifications of Domain Theory into other fields and what are the interactions with other fields? While ramifications can go just one way from Domain Theory into other fields, interactions are meant to go both ways.

Domain Theory cannot be compared to a field like Number Theory which has a long history, problems that one can explain to non-mathematicians and that is extremely rich with respect to the variety of methods that are used. Domain Theory can be compared to General Topology. General Topology provides a conceptual framework for phenomena that occur allover in Mathematics like *space, convergence, nearness, connectedness, compactness, etc.*, but it has not been developed in order to solve specific problems. Every theory has its internal life and creates its internal problems, but General Topology is mainly perceived as a background theory, and its latest internal developments may not matter to the outside world.

At the birth of Domain Theory, specific problems were waiting to be solved. Domain Theory provides a conceptual framework for phenomena occurring in various fields combining order and topology. It cannot be justified entirely by its internal problems. Thus, Domain Theory can only remain a lively research field if it interacts with other research areas inside or outside of Mathematics.

It is the purpose of this paper, firstly, to remind that historically the roots of Domain Theory have grown out of quite different origins, secondly, to indicate ramifications into other fields and interactions of Domain Theory with other fields that have occurred in the past and, thirdly, to draw the attention to a new development in a classical field of Mathematics, where Domain Theory appears as a relevant ingredient.

Because of the large variety of subjects to be touched, the paper will be far from being self-contained. It is hoped that the descriptions will be clear enough to give the flavor of things, so that one may be tempted to look at the original literature that is listed at the end of each of the three sections.

I apologize that my list of interactions will be incomplete and I would appreciate hints for making this paper more complete. What I write may be biased and can be criticized. And I hope to get lots of reactions.

The language of Domain Theory will be used freely. In particular, *poset* stands for partially ordered set, *dcpo* for directed complete poset, *domain* for continuous dcpo, $\ll$ denotes the way-below relation.

References are inserted at the end of each section, since they are of a different nature according to the very different contents of the three sections.