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Computing the Clique-width of Cactus Graphs

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Abstract

Similar to the tree-width (twd), the clique-width (cwd) is an invariant of graphs. A well known relationship between tree-width and clique-width is that $cwd(G) \leq 3 \cdot 2^{twd(G)-1}$. It is also known that tree-width of Cactus graphs is 2, therefore the clique-width for those graphs is smaller or equal than 6. In this paper, it is shown that the clique-width of Cactus graphs is smaller or equal to 4 and we present a polynomial time algorithm which computes exactly a 4-expression.

Keywords: Graph theory, Clique-width, Tree-width, Complexity.

1 Introduction

The clique-width has recently become an important graph invariant in parameterized complexity theory because measures the difficulty of decomposing a graph in a kind of tree-structure, and thus efficiently solve certain graph problems if the graph has clique-width at most k. A decomposition of a graph G, to compute its clique-width, can be viewed as a finite term, Courcelle et al. [5] define a term based on a set of four operations such as: 1) the creation of vertices, 2) disjoint union of graphs, 3) edge creation and 4) re-labelling of vertices. The number of labels (vertices) used to build the graph is commonly denoted by k. A well defined combination of these operations, called k-expression, are necessary to build the graphs, which in turn defines clique-width. The clique-width or the corresponding decomposition of the

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graph is measured by means of a k-expression [12]. As the clique-width increases the complexity of the respective graph problem to solve increases too, in fact for some automata that represent certain graph problems (according to the scheme in Courcelle's main theorem), computation runs out-of-memory, see [16] for some examples of graphs with the clique-width 3 or 4 .

It is important to look for an alternative graph decomposition that can be applied to a wider classes than to those of bounded tree-width and still preserve algorithmic properties. Tree decomposition and its tree-width parameter of a graph, are among the most commonly used concepts [7]. Therefore, Courcelle and Olariu proved that the clique-width can be seen as a generalization of tree-width in a sense that every graph class of bounded tree-width also have bounded clique-width [6].

In recent years, clique-width has been studied in different classes of graphs showing the behavior of this invariant under certain operations; the importance of the clique-width is that if a problem on graphs is bounded by this invariant it can be solved in linear time. For example Golumbic et al. [8] show that for every distance hereditary graph G, the $cwd(G) \leq 3$, so the following problems have linear time solution on the class of distance-hereditary graphs: minimum dominating set, minimum connected dominating set, minimum Steiner tree, maximum weighted clique, maximum weighted stable set, diameter, domatic number for fixed k, vertex cover, and $k-$colorability for fixed k. On the other hand the following graph classes and their complements are not of bounded clique–width: interval graphs, circle graphs, circular arc graphs, unit circular arc graphs, proper circular arc graphs, directed path graphs, undirected path graphs, comparability graphs, chordal graphs, and strongly chordal graphs [8].

Another major issue in graphs of bounded clique-width is to decide whether or not a graph has clique-width of size k, for fixed k. For graphs of bounded clique-width, it was shown in [3] that a polynomial time algorithm ($O(n^2m)$) exists that recognize graphs of clique-width less than or equal to three. However, as the authors pointed out the problem remains open for $k \geq 4$. On the other hand, it is well known a classification of graphs of clique-width ≤ 2, since the graphs of clique-width 2 are precisely the cographs. There are, however, some results in general. In [9] the behaviour of various graph operations on the clique-width are presented. For instance, for an arbitrary simple graph with n vertices the clique-width is at most $n-r$ if $2^r < n-r$ where r is rank [13]. In [10], it is shown that every graph of clique-width k which does not contain the complete bipartite graph $K_{n,n}$ for some $n > 1$ as a subgraph has tree-width at most $3k(n-1)-1$, whereas in [9] is shown that the clique-width under binary operations on graphs behaves as follows, if k_1, k_2 are the clique-width of graphs G_1, G_2, respectively, then $cwd(G_1 \oplus G_2) = \max(k_1, k_2)$, $cwd(G_1[v/G_2]) = \max(k_1, k_2)$ where $G_1[v/G_2]$ means substitute vertex v in G_1 by G_2. Similar results are presented for the *joint*, *composition*, *substitution* and some other important graph operation such as *edge contraction*, among others.

Regarding our present work, we are interested in the class of graphs, called *cactus*, which consist of non-edge intersecting fundamental cycles [11]. This class belongs to the class of bounded tree-width. These graphs have already a tree like