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On representing a simple polygon perceivable to a blind person

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ABSTRACT

Given the contour of a simple polygon P as an ordered set V of n vertices including a start vertex v, we model the optimization problem of representing P with a smallest-size unordered set $S = \{V \cup V'\}$ of vertices, where V' denotes an additional set of pseudo-vertices chosen along the edges of P such that P is perceivable uniquely by applying a progressive nearest-neighbor traversal rule. A traversal that uses the nearest-neighbor rule on the set S is said to perceive the polygon P if the traversal on S from the same start vertex $v \in S$ visits the vertices in P in the same order when the following rule is applied: Recursively choose the next nearest neighbor $v' \in S$ of v and then delete the last visited vertex v until all the vertices in S is traversed. The set S of vertices by itself should be tangible by touch (tactile information) in the sense that it is able to convey the perception of the shape to a blind reader in the same way as it was described in its input. A desirable objective in this context is to find the smallest-cardinality set V' such that P can be perceived uniquely from $S = \{V \cup V'\}$ using the nearest-neighbor traversal rule. In this paper, we propose to choose a set V^* with a sufficiently large cardinality such that the unordered set $S^* = \{V \cup V^*\}$ can be used to perceive P using the nearest-neighbor traversal rule. We also compute an upper bound on $|V^*|$ constructed by the proposed algorithm, in terms of certain geometric parameters of the polygon P.

1. Introduction

Our perceptual system may often find it difficult to reconstruct the shape of objects from sparse partial data (see Fig. 1). The recognition, perception, and discrimination of various geometric shapes become even more challenging for visually impaired persons. Blind persons identify alphabets through *Braille* characters, which are small rectangular cells that contain tiny palpable bumps called *raised dots*. The contour of a geometric shape can also be described by a sequence of labeled nodes akin to solving a *connect-the-dots* problem. However, in order to make geometric shapes perceptible to a blind person, a sequence of labeled nodes does not make any sense; on the other hand, a set of unordered (unlabeled) raised dots, chosen along the contour of the object in close succession, can offer a cost-effective solution to this problem. This *tactile map* of elevated dots, when felt by running fingers, provides a perception of the shape or content. If the elevated dots are placed close enough along an edge, the recognition of the original shape can be made unambiguously. One challenging problem in this context is to minimize the number of

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such raised dots or pseudo-vertices that are to be chosen along the contour. Compact representation of geometric shape is also a fundamental problem in many areas of computer vision, graphics, and document understanding. A detailed study on the perceptual saliency of points along an object-contour was made by Winter and Wasemans [6] from a psycho-visual perspective. In an earlier work Stelldinger [5] discussed the problem of human perception for recognizing curves given only point-information. We quote verbatim: *If the points are appropriately aligned, one simply "sees" the correct solution even if no further information is given*. There are also many situations where a set of sample points lying on or near a surface can be used to reconstruct a polygonal approximation of the surface [4].

Bhowmick et al. [3] first introduced the problem of representing the polygonal boundary of a digital object with an unordered set of vertices for image processing applications. An algorithm for recognizing the polygon was reported where the polygonal nodes are assumed to be in $\mathbb{Z}^2$. However, the problem in $\mathbb{R}^2$ was left open, and no finite bound is still known on the maximum number of such pseudo-vertices that are sufficient for reconstruction.

2. Problem description

We are given a simple polygon with an ordered set V of n vertices along with a start vertex $v \in V$. Our aim is to produce an unordered set $S = \{V \cup V'\}$, where V' denotes the set of pseudo-vertices (extra points) chosen along the edges of the polygon P.

Let S' be the sequence of vertices of S that is obtained starting from a given start vertex v_1 and by using the nearest-neighbor (NN) rule recursively. The NN-rule is formally described as Algorithm 1. The set of vertices in P in the final output, i.e., in S', should be stable with respect to the input. In other words, they should appear in the same order as they are given in the input.

Algorithm 1: Recursive procedure for computing S'.

Input: An unordered set of vertices S and a start vertex v_1
Output: An ordered set of vertices S'
Set $Flag(v_i) = 0$ for all i;
Let us define the unique nearest neighbor[a] of a vertex v as follows: $NN(v_i) = v_j \in S$, where v_j is the closest vertex of v where $Flag(v_j) = 0$;
$S' = \{v_1\}$;
$S = S \setminus v_1$;
$Flag(v_1) = 1$, $w = NN(v_1)$;
while *S is not empty* **do**
    $Flag(w) = 1$;
    $S = S \setminus w$;
    $S' = S' \cup \{w\}$;
    $w = NN(w)$;
Report S'

[a] NN(v) should have exactly one nearest neighbor of v.

Now we formally define the objective of our problem as follows:

Objective: Find a minimum-cardinality set V' that would allow us to perceive the polygon using the nearest-neighbor rule.

In this paper, we will show that there exists a finite subset V^* such that $S^* = V \cup V^*$ can be perceived by using the nearest-neighbor rule. We will describe an algorithm that outputs such a V^*. Although the set V^* may not be of smallest cardinality, (i.e., an optimal solution), we show that it is bounded by n, L and δ which are defined in Sec. 3.2.

It remains an open question to find the minimum cardinality set V' or to prove that finding V' is NP-hard.

We illustrate the problem of inserting pseudo-vertices on the original polygon with the help of an example. Consider the polygon shown in Fig. 1(a). The edge and labeling information of the vertices are removed in Fig. 1(b). Note that from this set of unlabeled vertices, many polygons can be drawn, two of which are shown in Figs. 1(c) and (d). However, when a few pseudo-vertices are inserted along the polygonal edges (refer to Fig. 1(e)), it is possible to recognize the given polygon uniquely (refer to Fig. 1(f)).

The rest of the paper is organized as follows: In Section 3, we have sketched the roadmap of our algorithm for inserting the pseudo-vertices along the edges of the polygon P, and described the algorithm for recognizing the original polygon P uniquely from S. The proposed method may need to handle certain degenerate cases; in Section 4, we have shown how to handle these degeneracies.

3. Roadmap of the proposed algorithm

In this section, we describe an algorithm to construct V^* such that $S^* = \{V \cup V^*\}$ can be used to perceive P using the nearest neighbor traversal rule. Let the vertices in the ordered set V be labeled as $v_1, v_2, \ldots, v_n$. Let $e_i = (v_{i-1}, v_i)$ denote the ith edge of P, and define $e_1 = (v_n, v_1)$. We add pseudo-vertices on each edge e_i in the increasing order of i. We now define certain useful geometrical parameters in the next subsection.

3.1. Geometric preliminaries

Let $V_{\mathcal{CH}}$ denote the convex hull of P. Every edge $e \in V_{\mathcal{CH}}$ that is not an edge of P, produces a pocket P_i (for $1 \le i \le k$), which is a simple polygon outside P but inside $V_{\mathcal{CH}}$. Each pocket P_i is defined by the edge e and a sequence of consecutive edges of P where the first and the last edge of the sequence are incident on the two endpoints of e. We construct a triangulation of P, and for all pockets $P_1, P_2, \cdots, P_k$ using any triangulation algorithm (see [2]) (refer to Fig. 2). We denote the resulting triangulated region as $\mathcal{T}_{\mathcal{CH}}$.

Define $dist(a, e_j)$ (see Fig. 3) as the minimum distance from a point a to an edge e_j. Let $|e_j|$ denote the length of the edge-segment e_j.

3.2. Insertion phase

The pseudo-vertices are inserted on each edge of P in $\mathcal{T}_{\mathcal{CH}}$ using a similar technique of inserting Steiner vertices on the edges [1] with certain minor modifications. Let v be a vertex of $\mathcal{T}_{\mathcal{CH}}$. Let $T_1^v, T_2^v, \ldots, T_k^v$ denote the set of triangles in $\mathcal{T}_{\mathcal{CH}}$ where v is one of the vertices in each of the triangles T_i^v $(i = 1, \ldots, k)$ and $l(T_i^v)$ is the edge