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Droplet dynamics in confinement

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ABSTRACT

This study is to understand confinement effect on the dynamical behaviour of a droplet immersed in an immiscible liquid subjected to a simple shear flow. The lattice Boltzmann method, which uses a forcing term and a recolouring algorithm to realize the interfacial tension effect and phase separation respectively, is adopted to systematically study droplet deformation and breakup in confined conditions. The effects of capillary number, viscosity ratio of the droplet to the carrier liquid, and confinement ratio are studied. The simulation results are compared against the theoretical predictions, experimental and numerical data available in literature. We find that increasing confinement ratio will enhance deformation, and the maximum deformation occurs at the viscosity ratio of unity. The droplet is found to orient more towards the flow direction with increasing viscosity ratio or confinement ratio. Also, it is noticed that the wall effect becomes more significant for the confinement ratios larger than 0.4. Finally, the critical capillary number, above which the droplet breakup occurs, is found to be mildly affected by the confinement for the viscosity ratio of unity. Upon increasing the confinement ratio, the critical capillary number increases for the viscosity ratios less than unity, but decreases for the viscosity ratios more than unity.

1. Introduction

Emulsions consist of immiscible fluids commonly found in production processes in food, chemical, and pharmaceutical industries. Since the droplet size and shape determine important emulsion properties such as stability, rheology, and particle morphology, it is important to understand the mechanism of droplet deformation and breakup during emulsification. In addition, the study of droplet deformation and breakup can provide valuable insights into immiscible fluid displacement in porous media, which plays an important role in enhanced oil recovery, geologic CO_2 sequestration, and remediation of nonaqueous-phase liquids. In recent years, the droplet deformation and breakup have received more attention because of the growing interest in microfluidic technologies, where droplets are circulated in channels and their size is often comparable with or even smaller than channel dimension. A significant number of theoretical, experimental and numerical studies have been reported regarding droplet deformation and breakup in a shear flow since the pioneering work of Taylor [1]. The main feature of shear flow is its relative simplicity, while it contains rich physics [2].

As sketched in Fig. 1, a spherical droplet with radius R is initially placed halfway between two parallel plates that are separated by a distance H. Shear flow is introduced to this system by the movement of the plates with equal but opposite velocities U, and the resulting shear rate is $\dot{\gamma} = 2U/H$. The droplet and the carrier fluid are assumed to have equal densities ρ, a constant interfacial tension σ, and their dynamic viscosities are μ_d and μ_m. In the simple shear flow, two most important forces exerting on the droplet are: the viscous forces ($\mu_m \dot{\gamma} R^2$), leading the droplet to deform and turn towards the flow direction, and the capillary forces (σR), resisting the droplet deformation and retaining the spherical shape of the droplet. Consequently, the droplet undergoes an elongation in the direction of the L-axis, and a compression in the direction of the B-axis (see the right panel of Fig. 1). The capillary number, which is defined as $Ca = \dot{\gamma} R \mu_m / \sigma$, is used to measure the relative magnitude of viscous and capillary forces. The inertial force can also influence the dynamical behaviour of the droplet, and its importance is described by the Reynolds number, defined as the ratio of inertial to viscous forces i.e. $Re = \rho \dot{\gamma} R^2 / \mu_m$. In addition to the capillary and Reynolds numbers, the viscosity ratio ($\lambda = \mu_d / \mu_m$) and the proximity of droplet to the walls are found to affect strongly the deformation and breakup of the droplet. The proximity is evaluated by the confinement ratio, defined as $2R/H$. Under consideration of low capillary number and confinement ratio, the droplet will eventually become ellipsoidal in the Stokes flow regime ($Re < 1$). The

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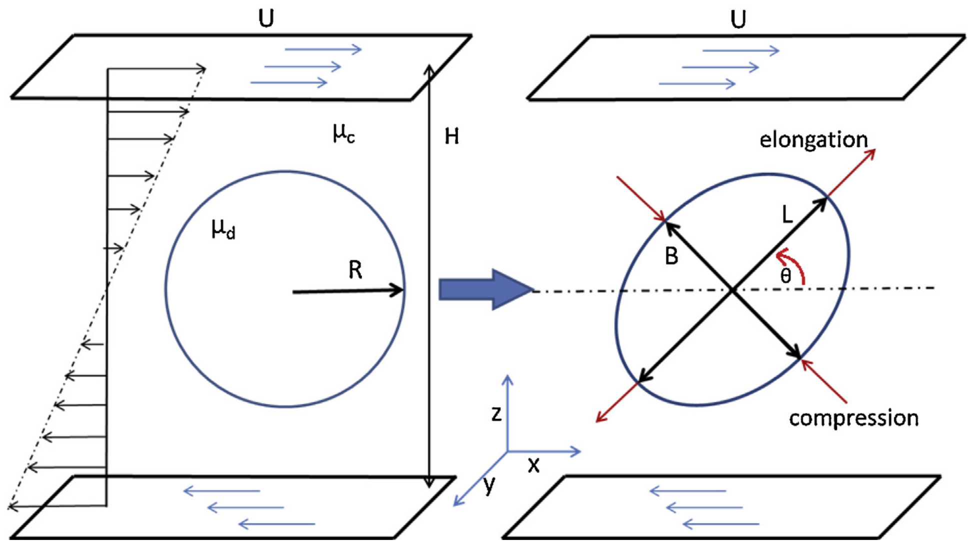

Fig. 1. Schematic illustration of droplet deformation in a simple shear flow.

deformation parameter D can be defined by the lengths of the major (L) and minor (B) axes of the deformed droplet, i.e.

$$D = \frac{L-B}{L+B}. \tag{1}$$

With increasing Ca, the droplet can reach a steady state but deviate from the ellipsoidal shape. This causes difficulty in obtaining the correct values of deformation parameter and orientation angle (θ), which is defined as the angle between the droplet major axis and the horizontal plane. As the capillary number is further increased above the critical value, the capillary forces can no longer retain the shape of droplet; the dominant viscous forces lead the droplet to form a long thin neck and finally break up into two or more fragments. The critical capillary number (Ca_{cr}), above which the droplet breakup will occur, is influenced by both the viscosity and confinement ratios [3].

The interest in droplet deformation dates back to the work of Taylor (1934) [1], who derived a theoretical expression to describe small deformations in the bulk shear flow in terms of the viscosity ratio and the capillary number:

$$D_T = \frac{19\lambda + 16}{16\lambda + 16} Ca. \tag{2}$$

This expression has been demonstrated to predict experimental results well in a variety of cases, where the value of confinement ratio was around 0.2. However, the influence of wall confinement is not taken into account in Eq. (2). It was reported that the presence of walls has a negligible contribution to the droplet deformation for the confinement ratio $2R/H \leq 0.4$ [4,5]. When the confinement ratio is higher than 0.4, the deformation cannot be predicted accurately by Eq. (2). Moreover, Eq. (2) is not able to describe the droplet deformation for very large viscosity ratios [1,6]. More discussion about this equation can be found in these review papers [7–10]. Taylor derived his model using small deformation perturbation procedure to the first order, with Ca as the expansion parameter, so he obtained a constant orientation angle of 45°. The perturbation procedure was later to be extended to the second order in Ca to yield an expression for the orientation angle [11–14]: $\theta = (\pi/4) - ((16+19\lambda)(3+2\lambda)/80(1+\lambda))Ca$ [15]. The orientation angle was also formulated differently in the phenomenological models of Maffetone and Minale [16] and Minale [17].

To address the wall confinement effect on droplet behaviour, Shapira and Haber [18] solved the Stokes flows around a droplet using the method of reflection, which takes into account the relative position of the droplet to the wall. The resulting SH model combines the Taylor deformation and an additional term accounting for the influence of walls on the deformation:

$$D_{SH} = D_T \left[1 + C_S \frac{1+2.5\lambda}{1+\lambda} \left(\frac{R}{H} \right)^3 \right], \tag{3}$$

where C_S is referred to as a shape parameter and its value depends on the relative position of the droplet to the walls. For a droplet positioned halfway between the two walls C_S is taken as 5.6996.

To predict the droplet deformation under transient conditions, Maffettone and Minale [16] proposed a phenomenological model, i.e. MM model, in which the droplet shape is assumed to remain ellipsoidal. A second order tensor $\mathcal{S}$, whose eigenvalues reflect the squares of the semi-axes of an ellipsoid, is used to describe the droplet shape. Based on this assumption, they derived an evolution equation for the tensor $\mathcal{S}$, which consists of a co-rotational derivative, the contributions of the viscous stress and the capillary force. The values of the semi-axes for several typical flows were predicted, including the simple shear flow, the uniaxial extensional flow and the planar hyperbolic flow. In particular, for the simple shear flow, the deformation parameter in a steady state is calculated as

$$D_{MM} = \frac{\sqrt{m_1^2 + Ca^2} - \sqrt{m_1^2 + (1-m_2^2)Ca^2}}{m_2 Ca}, \tag{4}$$

where

$$m_1 = \frac{40(\lambda+1)}{(2\lambda+3)(19\lambda+16)}, \tag{5}$$

and

$$m_2 = \frac{5}{2\lambda+3} + \frac{3Ca^2}{2+6Ca^2}. \tag{6}$$

The droplet deformation was also investigated experimentally by Sibillo et al. [19] for the viscosity ratio of unity. Three different capillary numbers, i.e. 0.1, 0.2, and 0.3, were studied for the confinement ratios ranging from 0.14 to 1.0. For the same capillary number, the droplet could obtain a more elongated shape for larger confinement ratios. Also, the droplet shape deviates from ellipsoid at very high confinement ratios, i.e., $2R/H \geq 0.8$. A similar study, for a range of viscosity ratios, was performed by Vananroye et al. [6]. The difference in the droplet deformation is not significant for different viscosity ratios at low confinement ratios. However, this difference grows considerably at large confinement ratios, and is more