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Mechanism design for aggregating energy consumption and quality of service in speed scaling scheduling ☆

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ABSTRACT

We consider a strategic game, where players submit jobs to a machine that executes all jobs in a way that minimizes energy while respecting the given deadlines. The energy consumption is then charged to the players in some way. Each player wants to minimize the sum of that charge and of their job's deadline multiplied by a priority weight. Two charging schemes are studied, the *proportional cost share* which does not always admit pure Nash equilibria, and the *marginal cost share*, which does always admit pure Nash equilibria, at the price of overcharging by a constant factor.

1. Introduction

In many computing systems, maximizing quality of service comes generally at the price of a high energy consumption. This is also the case for the speed scaling scheduling model considered in this paper. It has been introduced in [16], and triggered a lot of work on offline and online algorithms; see [1] for an overview.

The online and offline optimization problem for minimizing flow time while respecting a maximum energy consumption has been studied for the single machine setting in [15,2,6,9] and for the parallel machines setting in [3]. For the variant where an aggregation of energy and flow time is considered, polynomial approximation algorithms have been presented in [8,4,12].

In this paper we propose to study this problem from a different perspective, namely as a strategic game. In society many ecological problems are either addressed in a centralized manner, like forcing citizens to sort household waste, or in a decentralized manner, like tax incentives to enforce ecological behavior. This paper proposes incentives for a scheduling game, in form of an energy cost charging scheme.

Consider a scheduling problem for a single processor, that can run at variable speed, such as the modern microprocessors Intel SpeedStep, AMD PowerNow! or IBM EnergyScale. Higher speed means that jobs finish earlier at the price of a higher energy consumption. Each job has some workload, representing a number of instructions to execute, and a release time

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before which it cannot be scheduled. Every user submits a single job to a common processor, declaring the job parameters, together with a deadline, that the player chooses at his convenience.

The processor will schedule the submitted jobs preemptively, so that all release times and deadlines are respected and the overall energy usage is minimized. The energy consumed by the schedule needs to be charged to the users. The individual goal of each user is to minimize the sum of the energy cost share and of the requested weighted deadline. The weight is a private priority factor representing the individual importance of a small deadline. This factor includes implicitly a conversion factor that allows for an aggregation of the deadline and energy consumption into a single individual penalty.

In a companion paper [10] we study this game from the point of view of the game regulator, in a different setting. The players announce with their job their priority factors, and the regulator gets to decide on the completion time of the jobs. The usual questions one asks for such a game, is the existence of a cost sharing mechanism that would be truthful on the priority factors and which charge to the players amounts that sum up to a value comparable within a constant factor to the actual energy consumption of the schedule. This contrasts with the setting considered in this paper, where the player's strategies are the job's deadlines.

2. The model

Formally, we consider a non-cooperative game with n players and a regulator. The regulator manages the machine where the jobs are executed. Each player has a job i with a workload w_i, a release time r_i and a priority p_i, representing a quality of service coefficient. The player submits its job together with a deadline $d_i > r_i$ to the regulator. Workloads, release times and deadlines are public information known to all players, while quality of service coefficients can be private.

The regulator implements some *cost sharing mechanism*, which is known to all users. This mechanism defines a cost share function b_i specifying how much player i is charged. The penalty of player i is the sum of two values: his *energy cost share* $b_i(w, r, d)$ defined by the mechanism, where $w = (w_1, \dots, w_n), r = (r_1, \dots, r_n), d = (d_1, \dots, d_n)$ are the input values, and his *waiting cost*, which can be either $p_i d_i$ or $p_i(d_i - r_i)$; we use the former waiting cost throughout the article but all our results apply to both settings. The sum of all player's penalties, i.e., energy cost shares and waiting costs will be called the *utilitarian social cost*.

The regulator computes a minimum energy schedule for a single machine in the speed scaling model. In this model at any point in time t the processor can run at some speed $s(t) \geq 0$. As a result, for any time interval I, the workload executed in I is $\int_{t\in I} s(t)\,\mathrm{d}t$ at the price of an energy consumption valuated at $\int_{t\in I} s(t)^\alpha\,\mathrm{d}t$ for some fixed physical constant $\alpha \in [2, 3]$ which is device dependent [7].

The sum of the energy used by this optimum schedule and of all the players' waiting costs will be called the *effective social cost*.

The minimum energy schedule can be computed in time $O(n^2 \log n)$ [11] and has (among others) the following properties [16]. The jobs in the schedule are executed by preemptive earliest deadline first order (EDF), and the speed $s(t)$ at which they are processed is piecewise constant. Preemptive EDF means that at every time point among all jobs which are already released and not yet completed, the job with the smallest deadline is executed, using job indexes to break ties.

The cost sharing mechanism defines the game completely. Ideally, we would like the game and the mechanism to have the following properties.

existence of pure Nash equilibria This means that there is always a strategy profile vector d such that no player can unilaterally deviate from his strategy d_i while strictly decreasing his penalty.

budget balance The mechanism is c-budged balanced, when the sum of the cost shares is no smaller than the total energy consumption and no larger than c times the energy consumption. Ideally we would like c to be close to 1.

In the sequel we introduce and study two different cost sharing mechanisms, namely Proportional Cost Sharing where every player pays exactly the cost generated during the execution of his job, and Marginal Cost Sharing where every player pays the increase of energy cost generated by adding this player to the game.

3. Proportional cost sharing

The proportional cost sharing is the simplest budget balanced cost sharing scheme one can think of. Every player i is charged exactly the energy consumed during the execution of his job. Unfortunately this mechanism does not behave well as we show in Theorem 1.

Fact 1. *In a single player game, the player's penalty is minimized by the deadline*

$$r_1 + w_1(\alpha - 1)^{1/\alpha} p_1^{-1/\alpha}.$$