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Graph editing problems with extended regularity constraints

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ABSTRACT

Graph editing problems offer an interesting perspective on sub- and supergraph identification problems for a large variety of target properties. They have also attracted significant attention in recent years, particularly in the area of parameterized complexity as the problems have rich parameter ecologies.

In this paper we examine generalisations of the notion of editing a graph to obtain a regular subgraph. In particular we extend the notion of regularity to include two variants of edge-regularity along with the unifying constraint of strong regularity. We present a number of results, with the central observation that these problems retain the general complexity profile of their regularity-based inspiration: when the number of edits k and the maximum degree r are taken together as a combined parameter, the problems are tractable (i.e. in FPT), but are otherwise intractable.

We also examine variants of the basic editing to obtain a regular subgraph problem from the perspective of parameterizing by the treewidth of the input graph. In this case the treewidth of the input graph essentially becomes a limiting parameter on the natural $k+r$ parameterization.

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1. Introduction

Graph editing problems — problems where *editing operations* are applied to an input graph to obtain a graph with a given property — provide an interesting and flexible framework for considering many graph problems. For example virtually any problem whose witness structure is a subset of the vertices of the input graph can be alternatively phrased as a problem regarding the deletion of vertices to obtain a suitable property. k-Vertex Cover can be viewed as the problem of deleting at most k vertices such that the resultant graph has no edges. This target property can be viewed as a *degree constraint*; the degree of all vertices in the final graph should be zero. The nature of the resulting graph can also be defined by which editing operations are allowed; vertex deletion alone results in induced subgraphs, edge deletion alone produces spanning subgraphs, and so on.

Degree constraint editing problems have long been of interest to computational complexity theorists and this interest has been echoed in the parameterized complexity context. The proof of the NP-completeness of Cubic Subgraph was attributed to Chvátal by Garey and Johnson [16]. This naturally generalises to r-Regular Subgraph, which we can consider as the problem of removing vertices and edges to obtain an r-regular graph. r-Regular Subgraph is NP-complete [30] for $r \geq 3$, even under a number input constraints [6,33–35]. The problem of finding a maximum *induced* r-regular subgraph is NP-complete for $r \geq 0$ [5] ($r = 0$ is Maximum Independent Set and the removed vertices form a minimum vertex cover). If we allow only edge deletion, we have the r-Factor problem. For $r = 1$ this is the basic matching problem, well known to be

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polynomial [11,12,21,22]. If $r > 1$, or indeed if each vertex has a different target degree (the f-Factor problem), Tutte gives a reduction a polynomial-time solvable matching problem [37,38]. This problem can be further generalised by giving each vertex a range of target degrees (the Degree Constrained Subgraph problem) and is polynomial-time solvable [39]. If the edges of the graph have capacities (the (Perfect) b-Matching problem), the problem remains in P, using Tutte's f-Factor algorithm [20]. In the General Factor [23,24] problem allows each vertex to have a list of target degrees. If the lists contain gaps of greater than 1, the problem is NP-complete and polynomial-time solvable otherwise [7]. The problem of adding at most 2 vertices and a minimum number of edges to obtain a Δ-regular supergraph, where Δ is the maximum degree of the input graph, is polynomial-time solvable [2].

In the parameterized complexity setting, deleting k vertices to obtain an r regular graph is W[1]-hard for $r \geq 0$ with parameter k [26], but FPT with parameter $k+r$ [27]. Mathieson and Szeider [26] give a series of results for similar problems, which is extended further in Mathieson's doctoral thesis [25]. In this they examine the Weighted Degree Constrained Editing (WDCE) problem where the vertices and edges are weighted and each vertex has a set of target degrees, for combinations of vertex deletion, edge deletion and edge addition. When parameterized by the number k of edits allowed, the problem is W[1]-hard, when parameterized by the number of edits and the maximum value r in any of the degree lists, the problem is FPT. When the target degree sets are singletons and the editing operations include only edge deletion and addition, the problem is in P. The problem remains W[1]-hard with parameter k even in the unweighted case where each vertex has the same target degree r. In the singleton case, where vertex deletion or vertex deletion and edge deletion is allowed, the problem has a kernel of size polynomial in $k+r$ (and hence is in FPT for parameter $k+r$). The more general WDCE problem with vertex deletion and/or edge deletion has a kernel of size exponential in $k+r$, and hence is FPT for parameter $k+r$. Froese et al. [15] prove that no polynomial kernel is possible in these cases unless $\mathsf{NP} \subseteq \mathsf{coNP}/poly$. For the general weighted case with degree lists where the editing operations are any combination of vertex deletion, edge deletion and edge addition Mathieson and Szeider [26] give a logic based proof of FPT membership. Mathieson [25] shows that if vertex deletion and edge addition are allowed (and perhaps edge deletion), then in the weighted case (even with singleton vertex lists), no polynomial kernel is possible unless $\mathsf{NP} \subseteq \mathsf{coNP}/poly$. In particular they give the following central theorem:

Theorem 1 *(Mathieson and Szeider [26] Theorem 1.1). For all non-empty subsets S of* $\{\mathsf{v}, \mathsf{e}, \mathsf{a}\}$ *the problem* WDCE(S) *is fixed-parameter tractable for parameter $k+r$, and* W[1]*-hard for parameter k. If $\mathsf{v} \in S$ then $WDCE(S)$ remains* W[1]*-hard for parameter k even when all degree lists are restricted to $\{r\}$ and all vertices and edges have unit weight* 1.

Golovach [18] gives a concrete FPT algorithm for the unweighted case with parameter $k+r$ where vertex deletion and edge addition are allowed and shows that this case has no polynomial kernel unless $\mathsf{NP} \subseteq \mathsf{coNP}/poly$. Froese et al., in addition to the results mentioned above, show that the unweighted case with degree lists and edge addition has a kernel of size polynomial in $k+r$. In fact they show that either the instance is polynomial-time solvable or the kernel is polynomially-sized in r alone. Golovach [17] looks at the case where the target graph must also remain connected. Dabrowski et al. [8] look at the case where the input is planar and vertex deletion and edge deletion are allowed, and show that although still NP-complete, a kernel polynomially-sized in the number of deletions is obtainable. Belmonte et al. [1] study the problem of using edge contraction to fulfil degree constraints.

More recently, Bulian and Dawar [4] demonstrated a powerful meta-theorem based approach showing (amongst other results) that for a large collection of classes $\mathcal{C}$ of sparse graphs, determining the edit distance of an input graph G from some graph in $\mathcal{C}$ is fixed-parameter tractable. This meta-theorem gives many (if not all) of the results discussed here as a corollary, with the obvious caveat being essentially non-constructive in algorithmic terms.

1.1. Our contribution

In this paper we look at problems with alternative forms of degree constraints: *edge-degree-regularity*, *edge-regularity* and *strong-regularity* (q.v. Section 2 for definitions). We show that for these constraints, and with any combination of vertex deletion, edge deletion and edge addition, these problems are typically fixed-parameter tractable with the combined parameter $k+r$, para-NP-complete with parameter r and W[1]-hard with parameter k.

We also consider the parameterization of certain WDCE problems by the treewidth of the input graph where the number of editing operations is unbounded and show that finding an (induced) r-regular subgraph of graphs of bounded treewidth is in FPT. When both vertex deletion and edge addition is allowed, the problem becomes trivially polynomial-time solvable by simply editing the graph into a K_{r+1} if possible, and answering no otherwise.

2. Definitions and notation

We denote the closed (integer) interval from a to b by $[a, b]$. If $a = 0$, we denote the interval $[0, b]$ by $[b]$. We denote the power set of a set X by $\mathcal{P}(X)$.

In this paper we consider only simple, undirected graphs. Given a graph $G = (V, E)$ and two vertices u and v we denote the edge $\{u, v\} \in E$ by uv or vu. The *open neighbourhood* $N_G(u)$ of a vertex u is the set $\{v \mid uv \in E\}$. The *closed neighbourhood* $N_G[u]$ of a vertex u is $N_G(u) \cup \{u\}$. The degree of a vertex u is denoted $d(u)$ and $d(u) = |N_G(u)|$. Given an edge $uv \in E$, the *edge-degree* $d_G(uv)$ is the sum of the degrees of u and v, i.e. $d_G(uv) = d_G(u) + d_G(v)$. A graph is *$r$-regular* if for all $u \in V$

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