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Improved approximation algorithms for some min-max and minimum cycle cover problems

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ABSTRACT

Given an undirected weighted graph $G = (V, E)$, a set $\{C_1, C_2, \ldots, C_k\}$ of cycles is called a *cycle cover* of the vertex subset V' if $V' \subseteq \cup_{i=1}^{k} V(C_i)$ and its cost is given by the maximum weight of the cycles. The Min-Max Cycle Cover Problem (MMCCP) is to find a minimum cost cycle cover of V with at most k cycles. The Rooted Min-Max Cycle Cover Problem (RMMCCP) is to find a minimum cost cycle cover of $V \setminus D$ with at most k cycles, each of which contains one vertex in D. The Minimum Cycle Cover Problem (MCCP) aims to find a cycle cover of V of cost at most λ with the minimum number of cycles. We propose approximation algorithms for MMCCP and RMMCCP with performance ratios 5 and 6, respectively. These results improve the previous algorithms in term of both approximation ratios and running times. For MCCP we obtain a $\frac{14}{3}$-approximation algorithm that has the same time complexity as the previous best 5-approximation algorithm. Moreover, we transform a ρ-approximation algorithm for TSP into approximation algorithms for MMCCP, RMMCCP and MCCP with ratios 4ρ, $4\rho + 1$ and 4ρ, respectively.

1. Introduction

In the last two decades, considerable research attention has been devoted to the following fundamental vehicle routing problem. Given a fleet of k vehicles and a network, there is exactly one customer located at each vertex. Each vehicle has to start from some vertex to visit some customers and return to the same vertex. The travel costs between the vertices obey the triangle inequalities. The goal is to find a routing for the vehicles to collectively visit all the customers such that the maximum traveling cost of the vehicles is minimized. If described by graph theoretic language, the above problem is to cover all the vertices of an undirected weighted graph with at most k cycles such that the maximum weight of the cycles is minimum. It is called the Min-Max Cycle Cover Problem (MMCCP) in the literature (see [17]). In the rooted version, called the Rooted Min-Max Cycle Cover Problem (RMMCCP), the objective is to use at most k rooted cycles, i.e., cycles that each contains one vertex of a given depot vertices set, to cover the non-depot vertices such that the maximum weight of the cycles is minimum. In MMCCP and RMMCCP, if an upper bound $\lambda > 0$ is given for the weight of each cycle and the goal is to minimize the number of cycles used to cover the vertices, we obtain the Minimum Cycle Cover Problem (MCCP) and the Rooted Minimum Cycle Cover Problem (RMCCP), respectively.

The above-mentioned vehicle routing problems and their variants find numerous applications in operations research and computer science. RMMCCP and MMCCP were introduced by Even et al. [6] to model Nurse Station Location Problem. Camp-

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bell et al. [4] illustrated how disaster relief efforts can be improved by efficient algorithms for min-max cycle/path cover problems. Xu et al. [17] described some applications of cycle cover problems in wireless sensor networks. For more practical examples involving min-max and minimum vehicle routing problems we refer to [1,3,18,19,16,20] and the references therein.

Unfortunately, RMMCCP, MMCCP, RMCCP, MCCP are all NP-hard since they are extensions of the well-known Traveling Salesman Problem. Therefore, previous results mainly focus on devising approximation algorithms with good performance ratios.

1.1. Previous works

Xu et al. [20] showed that both MMCCP and RMMCCP cannot be approximated within performance ratio 4/3, unless P = NP. Xu and Wen [18] gave an inapproximability bound of 20/17 for the single-depot RMMCCP. By the NP-completeness of the well-known Hamiltonian Cycle Problem, both MCCP and RMCCP cannot be approximated within ratio 2.

A closely related problem, called the Min-Max Tree Cover Problem (MMTCP), can be obtained from MMCCP by replacing cycles with trees. The optimal value of MMTCP cannot be greater than that of MMCCP. But on the other hand, by duplicating each edge of a feasible solution of MMTCP we can obtain a feasible solution of MMCCP with the objective value doubled. Therefore, any α-approximation algorithm for MMTCP implies a 2α-approximation algorithm for MMCCP. Even et al. [6] and Arkin et al. [1] developed independently 4-approximation algorithms for MMTCP by different algorithmic techniques, and Khani and Salavatipour [12] give an improved 3-approximation algorithm. So MMCCP has a 6-approximation algorithm. Xu et al. [20] also derived an alternate 6-approximation algorithm. These algorithms were improved to a 16/3-approximation algorithm by Xu et al. [17] and Jorati [10] independently. In addition, Farbstein and Levin [7] showed that the algorithms for MMTCP and MMCCP can be improved further if the number k of vehicles is a fixed parameter instead of a part of the input.

For RMMCCP, Xu et al. [20] proposed a 7-approximation algorithm. Later, Xu et al. [17] improved the approximation ratio to 19/3. For the single-depot RMMCCP, Frederickson et al. [8] achieved a better ratio of $\rho + 1$, where ρ is the approximation ratio of the best available algorithm for the Traveling Salesman Problem. By using the well-known Christofides' Algorithm [5] this implies a 5/2-approximation algorithm. Moreover, Nagamochi [13], Nagamochi and Okada [14,15] obtained better results on a special case of RMMCCP where the graph is the metric closure of a tree.

By replacing cycles with trees in MCCP, we derive the Minimum Tree Cover Problem (MTCP), which is also named as the Bounded Tree Cover Problem in [12]. Since a cycle of weight at most λ can be split into two paths (which are also trees) of weight at most $\frac{\lambda}{2}$, two times the optimal value of MCCP cannot be less than the optimal value of MTCP with the upper bound $\frac{\lambda}{2}$ on the weight of the trees. On the other hand, by doubling each edge of a feasible solution of MTCP with the bound $\frac{\lambda}{2}$ we obtain a feasible solution of MCCP with the bound λ. Therefore, any α-approximation algorithm for MTCP implies a 2α-approximation algorithm for MCCP. Arkin et al. [1] developed a 6-approximation algorithm for MCCP. This conclusion is also implied by the 3-approximation algorithm for MTCP in the same paper. Khani and Salavatipour [12] gave an improved 5/2-approximation algorithm for MTCP, which indicates a 5-approximation algorithm for MCCP.

The approximability of RMCCP is far less understood. All the existing results focus on the case of one single depot vertex, which is called the Distance Constrained Vehicle Routing Problem by Nagarajan and Ravi [16]. The authors proposed a $\min\{\log n, \log\lambda\}$-approximation algorithm for the general case and a 2-approximation algorithm for the problem defined on the metric closure of a tree. Recently, Friggstad and Swamy [9] obtained an improved $\frac{\log\lambda}{\log\log\lambda}$-approximation algorithm for the general one single depot case.

1.2. Our results and techniques

In this paper we focus on MMCCP, RMMCCP and MCCP, and the main contributions are as follows. Firstly, we obtain a 5-approximation algorithm for MMCCP, which improves the previous best 16/3-approximation algorithm by Xu et al. [17] and Jorati [10]. Meanwhile, this algorithm also improves the running time from $O(n^5 \log \sum_{e\in E} w(e))$ to $O(n^3 \log \sum_{e\in E} w(e))$, where n is the number of vertices, E is the edge set of the graph and $w(e)$ is the weight of edge e. Moreover, we transform a ρ-approximation algorithm for TSP into a 4ρ-approximation algorithm for MMCCP, which implies further improvement on the performance ratio for the problem defined on some special metrics (e.g. Euclidean metric). Secondly, we show that any α-approximation algorithm for MMCCP implies an $(\alpha + 1)$-approximation algorithm for RMMCCP. This indicates a 6-approximation algorithm for RMMCCP, beating the 19/3-approximation algorithm in [17] in term of both performance ratio and running time. Thirdly, we devise a 14/3-approximation algorithm for MCCP with running time $O(n^5)$, improving the previous best 5-approximation algorithm by Khani and Salavatipour [12] with the same time complexity. Lastly, we introduce a new matching-based upper bound analysis for MMCCP which proves to be more efficient than the strategy of doubling tree edges in the literature.

The rest of the paper is organized as follows. We formally state the problems and give some preliminary results in Section 2, and deal with MMCCP, RMMCCP and MCCP in Sections 3, 4 and 5, respectively.

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