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The approximation algorithms for a class of multiple-choice problem

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ABSTRACT

Given a set of n points, which is a unit set of m colored sets, we study the minimum circle to cover one of each colored set at least. In this paper, we study some problems of optimizing some properties of color-spanning set. Firstly, we show a way to find a color-spanning set for each point and constitute a union of color-spanning sets without FCVD (The Farthest Color Voronoi Diagram). The approach to find each color-spanning set is based on the nearest neighbor points which have different colors. For every color-spanning set, we can find a enclosing circle to cover all points of each color-spanning set, which is a good approximate for MDCS (The Minimum Diameter Color-Spanning Set) problem. We propose an approximation algorithm for the SECSC (The Smallest Color-Spanning Circle) problem with 2-factor approximation result. For $|P| = n$ and $|Q| = m$, the worst running time of our algorithm is $O(n^2)$. Although these results are not as good as previous results with FCVD, the performance of our algorithms in R^d is much better than others. Moreover, an approximation algorithm can solve problem of minimum perimeter of the color-spanning set faster with time of $O(n^2 + nm\log n)$ and ratio $\sqrt{6}$. This result improved the ratio with only a little cost of time complexity. At last, we give an example for the 2-center SECSC problem. A fast computation of 2-center enclosing circle is proposed with time of $O(n^2)$, but the ratio depends on the gap between the nearest distinct colored distance.

1. Introduction

Background. The multiple-choice problems arise from the research about minimum diameter covering problem of Arkin and Hassin [1]. Supposing a set V together with subsets $S_1, \ldots, S_m$ is given, the cover is a subset of V containing at least one representative from each subset. In this paper, we consider the problem dealing with covering color classes, which is related to a class of multiple-choice problems. This kind of problems has been studied extensively due to wide applications in many areas such as facility location, statistical clustering, pattern recognition and data management in terrains and wireless networks [2–4]. The cover is always measure by different type of regions.

Related works. The enclosing circle problem was originally posed by Sylvester [5]. The goal of this classic problem is to find the smallest circle which can enclose a given point set. Dobkin et al. [6] and Overmas et al. [7] treated the problems of

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finding minimum perimeter convex k-gons. Their $O(k^2 n \log n + k^5 n)$ algorithm result was improved to $O(n \log n + k^5 n)$ by Aggarwal et al. [8].

However, the multiple-choice problem should cover each given subset but not all given elements. An interesting various problem is proposed by T. Graf and K. Hinrichs [9] and Abellanas et al. [4] named it as "color-spanning set". Given a set of n points, which is a unit set of m colored sets, we study the minimum diameter of the set covering at least one of each colored set. Some papers focus on computing the smallest color-spanning regions (such as disk, rectangle, equilateral triangle, convex hull and so on) such that some properties of the color-spanning set are optimal. Abellanas et al. [4] showed an algorithm for smallest color-spanning objects of axis-parallel rectangle and the narrowest strip. In the research about FCVD (The Farthest Color Voronoi Diagram) by Abellanas et al. [10], they proposed the circle with minimum diameter can be computed with time of $O(nk)$ when $k \leq (n/2)$, after the FCVD is provided using time $O(n^2(k) \log k)$. Without help of FCVD, it costs $O((k^3) n \log n)$ to get the smallest disk to cover all colors.

Under the condition of $k \leq \frac{n}{2}$, Eppstein et al. [11] studied the smallest polygons and proposed a $O(n^2 \log n)$ time algorithms using the Farthest color Voronoi diagram. If the colored points are in general position, it is proved to be NP-hard for some problems. Rudolf Fleischer and Xiaoming Xu [12,13] show that the minimum diameter color-spanning set problem is NP-hard, and proposed that the smallest color-spanning disk is a good approximation for the minimum-diameter rainbow set with ratio 2 for any d-dimensional space.

Two constant factor approximation algorithms were proposed for the minimum perimeter of the convex hull by Wenqi Ju et al. [14]. And their algorithms cost time of $O(n^2 + nm \log m)$ and $O(\min\{n(n-m)^2, nm(n-m)\})$ with the ratio of π and $\sqrt{2}$ respectively. However, as well as some other variants of these problems are based on the smallest polygons but Morton et al. [15] proved that such k-gons might not exist for $k > 7$.

Some versions of this problem come up with changing some parameters of the problem, such as the number of colors [16], the number of members in each color set [17,18], the dimension of the graphic object [19,20].

Some others proposed versions of this problem with constraint were proposed (Kamrmakar et al. [21]) and solved by the polynomial time algorithms. Zhang et al. [22] showed a brute force algorithm with $O(n^k)$ time for the MDCS problem (The Minimum Diameter Color-Spanning Set).

Interestingly, the approach in paper of Eppstein et al. [23] for the smallest polytopes, like the group Steiner tree by Graf et al. [9] and Wenqi Ju et al. [14], is based on the nearest neighbors. In this paper, for each point, we consider the nearest neighbor point which has different color firstly. Then we can find a color-spanning set based on these nearest neighbors. Our purpose is to find the minimum diameter of these subsets.

Our results. In this paper, we study some problems of optimizing some properties of color-spanning set. Firstly, we show a way to find a color-spanning set for each point and constitute a union of color-spanning sets without FCVD. Then, for every color-spanning set, we can find an enclosing circle to cover all points of each color-spanning set. Therefore, an approximation algorithm to compute the minimum diameter of the convex hull with the solution of SECSC (Smallest Enclosing Color-Spanning Circle) is proposed with the time of $O(n^2)$ at most. The approximation ratios of these algorithms are less than 2. Although these results are seen not better than previous results, the performance of our algorithms in R^d is much better than others. Moreover, based on these works, an approximation algorithm can solve problem of minimum perimeter of the color-spanning set faster with time of $O(n^2 + nm \log n)$ and ratio $\sqrt{6}$. This result improved the ratio with only a little cost of time complexity.

At last, we give an example for the 2-center SECSC problem, which is aimed to find the solution that makes the radius of the larger one of these two enclosing circle is minimum. Based on the approximation above, a fast computation of 2-center enclosing circle is proposed with time of $O(n \log n)$, but the ratio of this approximation algorithm depends on the gap between the nearest distinct colored distance.

2. Preliminaries and definitions

2.1. The color-spanning set

Here, a point set $P = (p_1, p_2, ..., p_j, ..., p_n)$ of n points on a plane and a color set $Q = (q_1, q_2, ..., q_i, ..., q_m)$ of m colors are given. We assume one point has and only has one of the given m colors and the number of colors m is fixed. Therefore, two properties of each point are its position and its color.

Definition 1. Let us call a region color-spanning if it contains at least one point of each color. That is to say, a point set S can be a color-spanning set iff its points belong to P and its color set equal to Q.

We assume the Euclidean distance between arbitrary two points is given. Therefore, for arbitrary point, the distance to its adjoining points can been certain.

Problem definition. An illustrative way to describe our basic problem is as follows: given a point set of n points with m colors, where each point has and only has one of the m colors, and a query $a = (a_1, ..., a_i, ..., a_m)$, where a_i is a integer for each $1 \leq i \leq m$, the points in a color-spanning set comes from at least a_i points of each color i. The minimum problem of color-spanning set aims to optimal some properties of the color-spanning region to cover at least a_i $(1 \leq a_i < m)$ points