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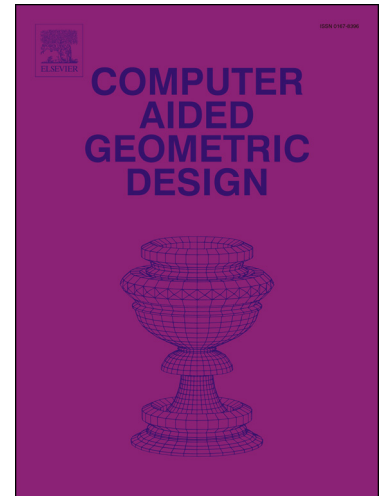
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Cubic Spline Approximation of a Circle with Maximal Smoothness and Accuracy

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Abstract

We construct cubic spline approximations of a circle which are four times continuously differentiable and converge with order six.

Keywords: Bézier curve, cubic spline, geometric smoothness, high accuracy approximation of circles

Mathematics Subject Classification: 41A10, 41A15, 68U05, 68U10, 65Dxx

Applying concepts of differential geometry to CAGD has led to a number of interesting results. An example is the surprising fact that spline curves can approximate with higher order than spline functions. In addition, smoothness constraints become less restrictive. A general statement to this effect is the following

Conjecture. For smooth curves in \mathbb{R}^d , which satisfy mild generic assumptions, there exist spline approximations of degree $\leq n$ which are $\alpha = n - 1 + \lfloor (n - 1)/(d - 1) \rfloor$ times continuously differentiable and converge with order $\alpha + 2$.

For low degree, many schemes of optimal order have been proposed (cf. [3, 7] and the references given there for an overview). However, except for the simplest case $n = d = 2$ treated in [6], the gain in accuracy has not yet been combined with a possible improvement of smoothness. This motivated this short note, which, aside from some obvious practical relevance, supports the conjecture for a canonical test case.

The construction of a, in some sense optimal, cubic spline approximation $t \mapsto p(t)$ of a circle is straightforward. We interpolate the circle at the points

$$P_k = (\cos(k\varphi), \sin(k\varphi)), \quad \varphi = 2\pi/m,$$

which separate the m cubic Bézier segments of p , and choose $k = 0, 1, \dots, m - 1$ as knots. Moreover, we match the tangents at P_k . For a completely symmetric configuration, this leaves the length of the tangent vectors as the only degree of freedom. In terms of a corresponding parameter δ , the interior control points of the curve segments are

$$P_k^\pm = P_k \pm \delta(-\sin(k\varphi), \cos(k\varphi)), \quad \delta = |p'(k)|/3.$$

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