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Compressed vibration modes of elastic bodies

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ABSTRACT

The natural vibration modes of deformable objects are a fundamental physical phenomenon. In this paper, we introduce *compressed vibration modes*, which, in contrast to the natural vibration modes, are localized ("sparse") deformations. The localization is achieved by augmenting the objective which has the vibration modes as minima by a L^1 term. As a result, the compressed modes form a compromise between localization and optimal energy efficiency of the deformations. We introduce a scheme for computing bases of compressed modes by solving sequences of convex optimization problems. Our experiments demonstrate that the resulting bases are well-suited for reduced-order shape deformation and for guiding the segmentation of objects into functional parts.

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1. Introduction

Vibration modes and their frequencies are fundamental for analyzing and simulating the dynamics of physical objects. In this work, we introduce an approach for the compression and localization of vibration modes of elastic bodies. The *compressed vibration modes* have a localized support while preserving properties of the natural vibration modes, *e.g.*, they form an orthonormal system in the space of configurations of an elastic body and the low-frequency modes correspond to low-energy deformations. The degree of localization can be controlled by a continuous parameter μ.

For applications, *e.g.*, for reduced-order simulations, the localization provides benefits. The vectors describing the compressed vibration modes of a discrete object are sparse, which results in less memory requirements for storing a basis and fewer arithmetic operations for adding or scaling the vectors. In our experiments, we used the compressed vibration modes for reduced-order simulation and deformation-based shape modeling. A second aspect is that the localization of the vibration modes adds a novel aspect to the modal analysis of deformable objects. Our experiments illustrate that the compressed modes localize in a structured way. We see potential in exploring this structure for modal and shape analysis and using it for applications. As a first step in this direction, we use the compressed modes for shape segmentation into functional parts.

To define the compressed vibration modes, we first characterize the natural vibration modes as the minimizers of an optimization problem, then we add a L^1 regularizing term to the objective to enforce compression. The idea of using L^1 regularization to localize modes of variational problems was introduced in Ozoliņš et al. (2013) and applied to Schrödinger's equation. The compressed vibration modes, we introduce, specialize this idea to the localization of modes of vibration of elastic bodies. There are significant differences in how we define and compute the compressed modes compared to Ozoliņš et al. (2013) and other work on the compression of modes like (Neumann et al., 2014; Boscaini et al., 2015; Kovnatsky et al., 2015). Whereas they compute all compressed modes in one optimization, we devise a reformulation of the L^1 regularized eigenproblem, which allows for computing the modes sequentially. This results in computation timings that scale linearly in the number of modes, whereas previous methods scale superlinearly. Moreover, to compute the modes, we

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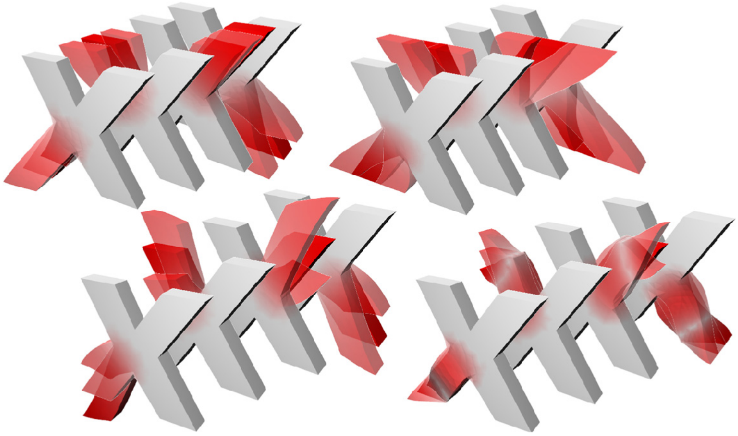

Fig. 1. The first sixteen compressed vibration modes of an X-shaped mesh ($\mu = 10$). We grouped the same modes (up to symmetry) for each of the four 'legs'.

devise an algorithm for solving the L^1 regularized optimization problem, which involves a convexification of a constraint as well as a linearization of the L^1 term. The algorithm allows to stably compute a large number of modes, as the runtime for each additional mode remains (almost) constant. The shown experiments demonstrate the benefits of this computational method over the ADMM method for the computation of compressed modes. An additional point is that our reformulation of the compressed eigenproblem allows for solving the L^1 constrained version of the problem. This formulation has the benefit over the L^1 regularized problem that the L^1 norm remains constant for all modes. This aspect has not been treated in prior work and seems difficult to achieve with prior problem formulations.

2. Related work

Vibration modes. Vibration modes are fundamental for describing, analyzing and simulating the dynamics of objects. The low-frequency vibration modes correspond to low-energy deformations of an object, which makes spaces spanned by low-frequency vibration modes attractive for reducing the computational cost of simulations and optimizations. In recent work in graphics, vibration modes are used for reduced-order models for deformation-based editing (Hildebrandt et al., 2011), deformable object simulation (Hauser et al., 2003; Barbič and James, 2005; von Tycowicz et al., 2013; Yang et al., 2015), sound synthesis (Chadwick et al., 2009; Li et al., 2015), and spacetime control of animations (Barbič et al., 2012; Hildebrandt et al., 2012a; Schulz et al., 2014; Li et al., 2014). Besides the dimensional reduction of simulations and optimizations, vibration modes are used for the analysis of objects (Hildebrandt et al., 2010, 2012b) and the segmentation of objects into functional parts (Huang et al., 2009). One drawback of using vibration modes for dimensional reduction is that the resulting basis vectors are dense, which can be problematic when a large basis is used or applications, like games, impose strict limits on the memory available for the reduced simulation. This problem has been addressed in Langlois et al. (2014) by applying a data compression scheme for storing the basis vectors. We present, for the first time, a localization of vibration modes, which allows for creating bases of low-energy deformations that are sparse. One resulting benefit for reduced-order methods is a reduced memory requirement.

L^1 regularization for eigenvalue problems. *Compressed modes for variation problems* were introduced by Ozoliņš et al. (2013) and used for computing localized bases for Schrödinger's equation. For the numerical computation of the compressed modes, they used a *splitting orthogonality constraint (SOC)* scheme. The approach was extended to the computation of compactly supported multiresolution bases for the Laplace operator on planar domains by Ozoliņš et al. (2014) and the sparse approximation of differential operators in the Fourier domain by Mackey et al. (2014). Compressed eigenfunctions of the Laplace–Beltrami operator of curved surfaces, called *compressed manifold modes*, were considered by Neumann et al. (2014) and used for mesh segmentation and functional correspondences. For computing the compressed manifolds modes, they proposed a scheme based on the alternating direction method of multipliers (ADMM, Boyd et al., 2011) and demonstrated that it outperforms the SOC scheme. Boscaini et al. (2015) used the compressed manifold modes for building class-specific descriptors for non-rigid shapes. Houston (2015) proposed a natural ordering for the compressed manifold modes along with an adaptation of the algorithm that is reported to significantly reduce the number of ADMM iterations required in the optimization. Since the definition of compressed modes includes a unit L^2 norm constraint, the feasible set for the optimization is not a linear space, but a curved manifold. A generic algorithm for L^1 regularized minimization problems over manifolds called the manifold alternating direction method of multipliers (MADMM) was introduced by Kovnatsky et al. (2015) and used for the computation of compressed manifold modes. Concurrent to our work, Bronstein et al. (2016) proposed a discretization of the L^1 norm for linear Lagrange finite elements on meshes and an iteratively reweighted least-squares scheme for computing compressed manifold modes. In this paper, we opt for a different approach for computing

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