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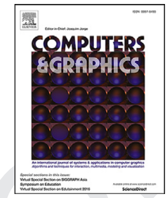

Technical Section

Embedding shapes with Green's functions for global shape matching

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ABSTRACT

We present a novel approach for the calculation of dense correspondences between non-isometric shapes. Our work builds on the well known functional map framework and investigates a novel embedding for the alignment of shapes. We therefore identify points with their Green's functions of the Laplace–Beltrami operator, and hence, embed shapes into their own function space. In our embedding the L_2 distances are known as the biharmonic distances, so that our embedding preserves the intrinsic distances on the shape. In the novel embedding each point-to-point map between two shapes becomes and can be represented as an affine map. Functional constraints and novel conformal constraints can be used to guide the matching process.

1. Introduction

Finding correspondences between two or more shapes is an important sub-task for a variety of applications, in which information has to be transferred or correlated between shapes. For example, local deformations can be transferred for shape editing [1–3], and correlations between corresponding regions can be exploited to compress dynamic meshes [4,5] and to create generative shape models [6–8].

Finding correspondences is especially interesting between non-isometric shapes, see Fig. 1 for some examples. Many previous approaches tailored to register isometric shapes fail in this case. Extrinsic non-rigid ICP [7,9] and variants [1,10–12] suffer from unreliable correspondences on extrinsic distances and from difficulties in solving the non-linear deformation models. The Blended Intrinsic Maps method [13,14] replaces the extrinsic metric by an intrinsic one and delivers good registration results by assuming the resulting maps to be locally conformal. A problem of BIM is that it is not clear how to incorporate a priori constraints which might be necessary to guide the method to the correct map out of the multiple reasonable ones (Fig. 2). Furthermore, the stitching of local maps leads to inconsistencies at their boundaries. Another group of approaches embeds shapes into a high dimensional space, where L_2 distances approximate intrinsic distances. Although most of them allow to incorporate additional constraints, many share the major drawback that their embedding requires a *non-linear* alignment.

Functional maps [15] overcome this problem by constructing an embedding, in which shapes can be aligned with a *linear* deformation. Unfortunately L_2 distances of delta-distributions, that are typically used to embed points, only approximate intrinsic distances between intrinsically close points, see Fig. 4. This is especially important when only a few implicit constraints are available, such as when matching non-isometric shapes.

In contrast to functional maps [15] we identify points with their Green's functions of the Laplace–Beltrami operator. In this embedding the L_2 distances are the well known biharmonic distances [16], which are an intrinsic distance metric on the shape. They are invariant to isometric shape deformations so that pose deformations have little influence on the matching process. We calculate correspondences by aligning these embeddings with an *affine* deformation, which can be computed reliably and efficiently. There is a linear relation between the Green's alignment and the (pull-back) functional map [15], so that we can incorporate functional constraints and operator commutativity into our setting. Last but not least, we can include additional constraints on the alignment, which require the resulting map to be close to conformal.

The main contributions of our paper are (a) a novel embedding of shapes in the functional map framework by identifying points with their Green's functions, (b) combining our novel embedding with functional constraints and (c) including conformality constraints into functional shape matching.

The paper is organized as follows: Section 2 discussed the related work. Section 3 introduces the alignment of shapes with Green's functions and relates it to the functional maps framework. We further motivate the novel embedding by a comparing Green's functions and delta-distributions in Section 4. Then Section 5 shows how to utilize functional constraints and

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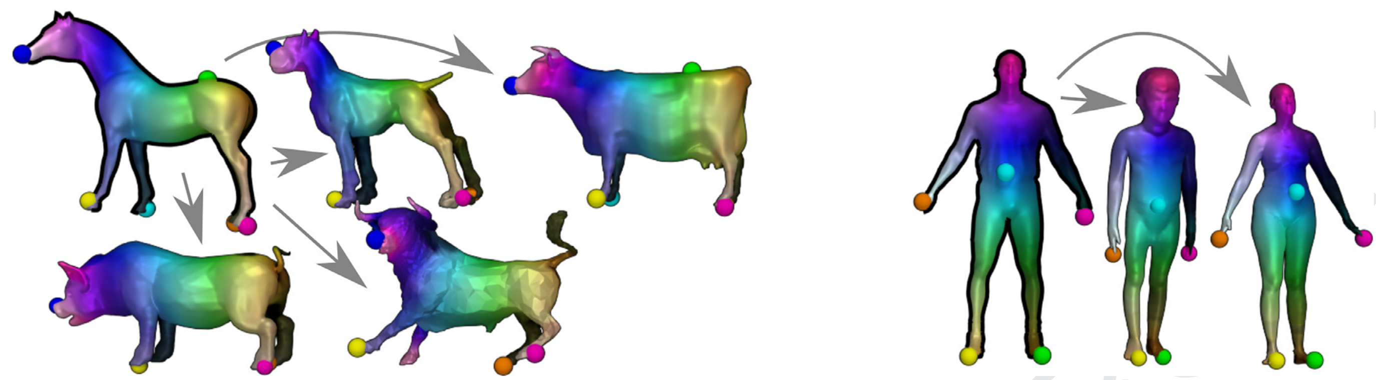

Fig. 1. A variety of results gathered with our method. For each class (fourlegged, teddy, humans, birds) there is a single source (black contour) and a variety of target shapes. Using the sparse correspondences depicted by small spheres we calculate a dense map from the source to each target shape, which we then use to transfer a color field. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

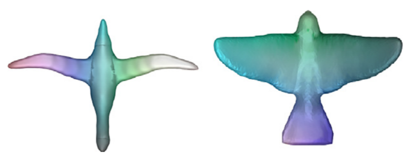

Fig. 2. Failure case of Blended Intrinsic Maps, that are difficult to resolve without predefined constraints.

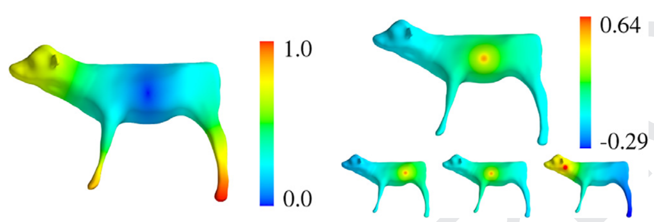

Fig. 3. (Left) Biharmonic distances $d_b(p, x)$ from a fixed point p and (right) its Green's function g_p and three other Green's functions.

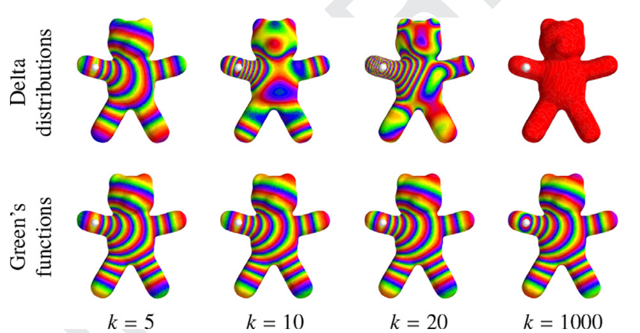

Fig. 4. Biharmonic distances and L_2 distances on delta-distributions for different numbers of eigenvectors.

conformality in the matching process. After discussing the discretization in Section 6 we describe a shape matching algorithm in Section 7, which is evaluated in Section 8.

2. Related work

Estimating correspondences between different shapes is a challenging task that has been addressed intensively in literature. In this section, we only provide a brief overview on directly related works and kindly refer the interested reader to the recent surveys [17,18].

2.1. ICP

Initially the problem of shape matching appeared in the context of registering sequential point cloud scans of a static scene. This led to the development of rigid ICP algorithms [19], which alternate between detecting corresponding points and rigidly aligning shapes. Due to the local optimization, these techniques depend strongly on the initial correspondences and on heuristics to prune novel correspondences. A variety of methods extend the original ICP metaphor to match deformed shapes by allowing non-rigid deformations in $\mathbb{R}^3$ for the alignment [1,7,9–12]. A common shortcoming of these methods is the detection of corresponding points based on extrinsic instead of intrinsic distances. For deformable shapes extrinsic distances can be small even for intrinsically distant points. As a consequence these methods typically require a large number of point-to-point constraints to begin with and utilize sophisticated heuristics to prune novel correspondences.

The Blended Intrinsic Maps method [13] obtains good results by concatenating and blending multiple conformal maps into a single global map. However, it cannot incorporate user constraints, which are sometimes necessary to solve ambiguities (e.g. Fig. 2). Furthermore, at the boundaries of the local maps the results often exhibit discontinuities. Additionally the method is difficult to generalize to point-clouds or shapes of genus other than zero.

Other methods map shapes by parameterizing them on a common domain and then aligning their parameterizations so that either an intrinsic measure of stretch from source to target becomes minimal [21–25] or so that the integrated stretch along a sequence of deformations from the source onto the target [26–28] becomes minimal. These methods deliver continuous maps of high quality, but are often computational demanding and their application on non-simple topologies is non-trivial. Q2

Yet another class of methods uses an ICP-like alignment *after* embedding shapes into a high dimensional space where L_2 distances approximate intrinsic ones. Shapes have been embedded with the eigenvectors of an affinity matrix [29,30], with an embedding approximating geodesic distances [31], with an embedding based on electrostatic repulsion [32] and with delta-functions [33]. All of these methods use non-linear maps to align the embeddings. Slightly different are the methods [34,35], where the alignment of shapes is avoided by directly embedding one shape into the other by minimizing a non-linear functional.

2.2. Functional maps

The remarkably successful functional maps framework was introduced in [15]. To the best of our knowledge this paper was the first to fully exploit the fact that a *linear* alignment of a

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