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Hole detection in a planar point set: An empty disk approach

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A B S T R A C T

Given a planar point set *S*, outer boundary detection (shape reconstruction) is an extensively studied problem whereas, inner boundary (hole) detection is not a well researched one, probably because detecting the presence of a hole itself is a difficult task. Nevertheless, hole detection has wide applications in areas such as face recognition, model retrieval and pattern recognition. We present a Delaunay triangulation based strategy to detect the presence of holes and an algorithm to reconstruct them. Our algorithm is a unified one which reconstructs holes, both for a boundary sample (points sampled only from the boundary of the object) as well as for a dot pattern (points sampled from the entire object). Our method is a non-parametric one which detects holes irrespective of its shape. Assuming a sampling model, we provide theoretical analysis of the proposed algorithm, which ensures the correctness of the reconstructed holes, for specific structures. We conduct both qualitative and quantitative comparisons with existing methods and demonstrate that our method is better or comparable with them. Experiments with varying point densities and distributions demonstrate that the algorithm is independent of sampling. We also discuss the limitations of the algorithm.

1. Introduction

Given a finite set of points, $S \subseteq \mathbb{R}^2$ (Fig. 1(a)), shape reconstruction problem asks for a shape in $\mathbb{R}^2$ that best approximates *S* [6]. Most of the existing works in shape reconstruction such as [5,18], focus only on outer boundary detection (Fig. 1(b)). The outer boundary can be considered as a convex/non-convex simple polygon, enclosing all points of *S* [6].

Visually, Fig. 1(c) captures the features of the shape better than Fig. 1(b), because of the presence of both outer and inner boundaries. An inner boundary (hole) can be considered as an empty convex/non-convex simple polygon which is enclosed within a boundary. Hole detection problem computes a best approximation of inner boundaries of *S*.

A planar point set can be classified into two types: (i) boundary sample [9] or curve sample [17] and (ii) dot pattern [9] or object sample [17]. If the points are sampled only from the boundary of the object, it is known as a boundary sample (BS), as shown in Fig. 2(a). If the points are sampled from the whole object, it is known as a dot pattern (DP), as shown in Fig. 2(b). Fig. 2(c) and (d) are the reconstructed shapes for the boundary sample and dot pattern, respectively and we denote the reconstruction of holes for BS and DP as RBS and RDP, respectively.

Given a point set *S* with its reconstructed outer boundary using Delaunay triangulation based methods such as [5,15,17], hole detection problem computes one of the best approximations of inner boundaries of *S*. From the set of triangles of the outer boundary reconstructed triangulation (output of [5,15,17]), we propose an algorithm to detect a triangle as the initial hole and expand it to obtain the hole boundary, based on the area and adjacency information of the triangle.

1.1. Motivation

We focus on the hole detection problem due to: (i) the challenges posed by the problem (ii) varied applications for the hole detection (iii) existence of only a few works addressing the problem and (iv) non-existence of a unified method for both reconstruction of boundary sample and reconstruction of dot pattern.

1.1.1. Challenges

The challenges associated with the outer boundary detection (such as ill-posed nature of the problem, dependence of the reconstructed output on density & distribution of the input point set, human cognition and perception [6]) exist for hole detection as well. Apart from them, another major challenge of hole detection is

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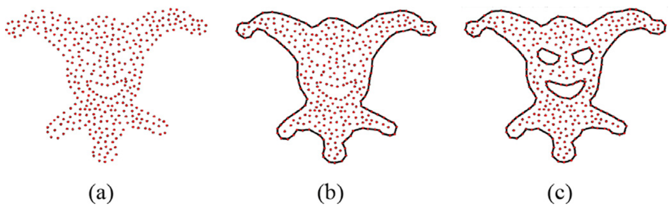

Fig. 1. (a) Input point set (b) Shape with reconstructed outer boundary (c) Shape with reconstructed outer and inner boundaries.

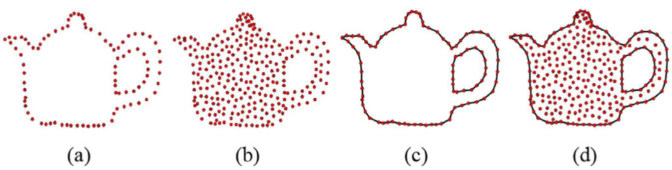

Fig. 2. (a) Boundary sample (b) Dot pattern (c) Reconstructed boundary sample (d) Reconstructed dot pattern.

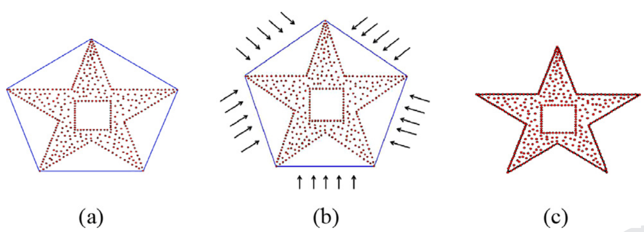

Fig. 3. (a) Convex hull (b) Shrinking of convex hull (c) Reconstructed outer boundary.

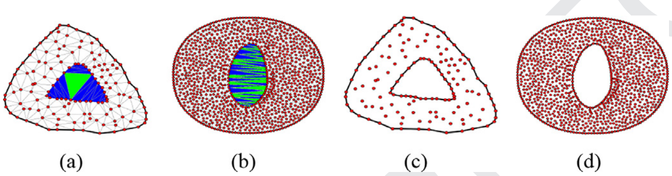

Fig. 4. (a) Hole structure specified in RGG [17]: A fat triangle (green in color) surrounded by a set of thin triangles (blue in color) (b) Hole without the structure specified in RGG (c)–(d) Reconstructed holes of different structures. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to identify the presence of a hole (initial guess of a candidate hole region to start the algorithm) in the given point set. In the case of outer boundary detection, a natural choice for the initial guess of the outer boundary is the convex hull (*CH*) of the point set (blue color boundary shown in Fig. 3(a)), because of the following reasons: (i) *CH* encloses all the points of *S* and (ii) all the vertices of *CH* are part of the reconstructed outer boundary. As an analogy, *CH* can be considered as a rubber band and the rubber band is shrunk (Fig. 3(b)) to compute the outer boundary as shown in Fig. 3(c). An analogous structure to that of the convex hull is not available for the hole detection. Hence, identifying the initial hole region to start with, is a challenging one.

1.1.2. Applications

Hole detection has various applications in fields such as face recognition, model retrieval, pattern recognition etc. Hole detection is used in face detection algorithms, where a hole mapping is used to detect certain facial characteristics such as mouth, nose, eyes and ears [19]. Hole detection has also applications in areas such as three-dimensional (3D) model retrieval system [12] and 2D point set matching. 3D point sets can be visualized by a collection of 2D views and it is easier to obtain the visual similarity between 2D point sets, if both inner and outer boundaries are reconstructed and thus the 3D model retrieval can be made more efficient. Specifically, Computer Aided Design (CAD) models are characterized by features like holes, tunnels, ribs and helixes [12]. Outer and inner boundary detected 2D point sets of CAD models make the matching more effective and accurate.

Other applications of hole detection are in the areas of Wireless Sensor Networks (WSNs) and power systems. Detecting the holes is a deciding factor for the efficiency of communication in WSNs [10]. Island (hole) formation in the power systems is a causality factor which has to be considered for the study of security analysis and control of power systems [11].

1.1.3. Related work

Unlike in the case of outer boundary detection problem, to the best of our knowledge, only a few works such as [2,4,7,8,13,14,16,17] exist for hole detection, perhaps because the latter problem is more challenging than the former. Most of the existing works are Delaunay triangulation based, except the one proposed in [8].

α-shape is the space generated by connecting point pairs that can be touched by an empty disk of radius α [7]. The points of weighted $\mathcal{A}$-shape [14] are the vertices of the Voronoi diagram and the centers of the Delaunay circle having radius greater than the specified threshold value, with weights associated with points in sparse regions. V. Kurlin proposed a method to compute number of holes from a given noisy point cloud, based on topological persistence [13]. The above methods are parametric, in which a parameter was tuned to obtain different outputs for the same input. Even though parameter tuning provides flexibility for the user to select the shape based on their requirements, it is very tedious to tune the parameter to obtain the best perceived shape.

Crawl through neighbors (crawl) [16], reconstruction of RGG (Relaxed Gabriel Graph - output of 2D reconstruction algorithm [17], which is a collection of most of the Gabriel edges and a few non-Gabriel edges induced by a Delaunay triangulation), crust [2], NN-crust [4] are non-parametric methods, which detect both outer and inner boundaries. Crawl is a Delaunay triangulation based method, designed for boundary sample. RGG is designed for handling dot pattern as input, which detects a hole only if the corresponding region in the point set has a structure in which a fat triangle is surrounded by a set of thin triangles as shown in Fig. 3(a). For a general point set, there is no guarantee that a hole region follows a particular structure, and hence the hole may be of any structure such as shown in Fig. 3(b)), whose reconstructed holes are shown in Figs. 3(c)–3(d). Hence, it is challenging to develop a non-parametric algorithm, which in practice, works irrespective of the structure of the hole. Crust and NN-crust are designed to work for boundary samples. An approximate positioning of network nodes near the hole boundaries was done in [8].

1.1.4. Unified method for reconstruction of RBS and RDP

Given an input point set, it is not easy to find out whether it is a boundary sample or dot pattern. Hence, apart from the challenges of reconstruction problem in general, there is a requirement for a unified method for hole detection, which works for both boundary sample and dot pattern. Simple shape [9], a parametric algorithm that works for both BS and DP, can reconstruct only outer boundary. The existing hole detection algorithms are either designed for RBS [2,4] or for RDP [17]. Hence, it is challenging to develop a unified algorithm for hole detection which works for both RBS and RDP. The primary motivation of a unified approach for hole detection is to provide an approach which is independent of the nature of the input.