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# Dynamic analysis and controller design for a slider–crank mechanism with piezoelectric actuators

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**Abstract**

Dynamic behaviour of a slider–crank mechanism associated with a smart flexible connecting rod is investigated. Effect of various mechanisms' parameters including crank length, flexibility of the connecting rod and the slider's mass on the dynamic behaviour is studied. Two control schemes are proposed for elastodynamic vibration suppression of the flexible connecting rod and also obtaining a constant angular velocity for the crank. The first scheme is based on feedback linearization approach and the second one is based on a sliding mode controller. The input signals are applied by an electric motor located at the crank ground joint, and two layers of piezoelectric film bonded to the top and bottom surfaces of the connecting rod. Both of the controllers successfully suppress the vibrations of the elastic linkage.


*Keywords:* Slider–crank mechanism; Flexible connecting rod; Piezoelectric; Dynamic response, Nonlinear controller

## 1. Introduction

High operating speed, superior reliability and accurate performance are major characteristics of modern industrial machinery and commercial equipments. A traditional rigid-body analysis, which presumes low operating speeds, becomes insufficient to describe the performance of such high speed systems. A thorough understanding of the dynamic behaviour of the modern machines undergoing high-speed operations, which are based on multibody systems such as slider–crank mechanisms, is necessary. Several researchers have worked on development of suitable formulations with these mechanisms. Neubauer et al. examined the transverse deflection of an elastic connecting rod of a slider–crank mechanism by neglecting the longitudinal deformation, the Coriolois, relative tangential and relative normal components of the acceleration [1]. Hsieh and Shaw studied the nonlinear resonance of a flexible connecting rod by considering both longitudinal and transverse deflection of the rod [2]. They investigated that the connecting rod behaves as a system with a softening type of nonlinearity, which is subjected to external and parametric excitations. Chen and Chian studied effect of crank length on the dynamic behaviour of damped flexible connecting rod [3]. Zheng et al. and Muvengi et al. have considered the effect of joint clearance and Reis et al added the effect of friction in dynamic analysis of the mechanism [4–6]. Complexity of the dynamic model of flexible mechanisms and their high nonlinearities make these systems hard to control. A few researchers have attempted to reduce or eliminate the vibrations of flexible mechanisms induced by one or more of the flexible links [7–9].

Karkoub and Yigit designed a controller for a four-bar mechanism with a flexible coupler. Their closed-loop system was able to trace a prescribed motion at the input link level. The PD controller was able to move the mechanism to the desired position and absorb the elastodynamic vibrations [10]. Karkoub has also developed a controller based on $\mu$ synthesis for suppressing the elastodynamic vibrations of a slider–crank mechanism associated with a very flexible connecting rod [11]. Sannah and Smaili designed a multivariable optimal controller for a four-bar mechanism with a flexible coupler using a finite

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**Nomenclature**

| | |
|---|---|
| $r$ | crank length |
| $L$ | connecting rod length |
| $\theta$ | crank angle |
| $\psi$ | connecting rod angle with respect to the ground |
| $q_i(t)$ | modes of vibrations of the flexible slider–crank mechanism |
| $F_i$ | nonconservative forces |
| $\tau_i$ | applied torque on the system |
| $\vec{\xi}$ | deflection vector |
| $Ic$ | moment of inertia of the crank |
| $A$ | cross section of the connecting rod |
| $Ms$ | slider mass |
| $Mc$ | crank mass |
| $EI$ | flexural rigidity |
| $\rho$ | material density |
| $H$ | radius of the rod |
| $d_{31}$ | dielectric coefficient |
| $V$ | applied voltage to piezoelement |
| $\vec{X}_B$ | velocity of the connecting rod end point |

element dynamics model. The results were implemented on an experimental test bed using a pair of piezoceramic sensors/actuators [12].

Here, we focus on studying effect of various mechanisms' parameters on the dynamic behaviour and rotation of the crank considering the transverse deflection of the connecting rod. Even with no external excitation, rotation of the crank excites the connecting rod and induces vibration. We successfully suppressed the vibrations of the elastic linkage using two piezoelectric actuators and nonlinear controllers designed based on feedback linearization and sliding mode.

## 2. Modeling of the mechanism

Equation of motion of a flexible slider–crank mechanism is derived using the Euler–Lagrange approach [13–17]. The mechanism is assumed to move in the horizontal plane and the longitudinal defections are negligible. Schematic of the slider–crank mechanism with a flexible connecting rod is depicted in Fig. 1. The mechanism parameters are defined as follows: $r$ is the crank length; $L$ is the connecting rod length; $\theta$ is the crank angle; $\psi$is the connecting rod angle with respect to the ground; $x$ and w are the $x$- and $y$-coordinates, respectively, of any point on the connecting rod in the $\vec{e}_1-\vec{e}_2$ coordinate system.

The location of any point on the flexible connecting rod (Fig. 1) is given by

$$\vec{R}=\vec{r}+\vec{x}+\vec{w} \tag{1}$$

equal to

$$\vec{R}=(r\cos\theta+w\cos\psi+x\cos\psi)\,\vec{i}+(r\sin\theta+w\sin\psi-x\sin\psi)\,\vec{j} \tag{2}$$

The $y$-component of the displacement of the end point of the connecting rod at $x=l$, which can be obtained by taking the scalar product of the displacement vector $\vec{R}$ and $\vec{j}$ is equal to zero. Therefore

$$\psi=\sin^{-1}\left(\frac{r}{l}\sin\theta\right) \tag{3}$$

Using the mode summation technique, the deflection $w$ is given by

$$w=\sum_{i=1}^{n}\sin\left(\frac{i\pi x}{l}\right)q_i \tag{4}$$

where $q_i(t)$ are the modes of vibrations of the flexible slider–crank mechanism. To derive the model for the flexible mechanism the Euler–Lagrange equations are used. Let $L=T-U$, where $T$ and $U$ are the kinetic and potential energies of the system, respectively. The equations of motion can be obtained using the following equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\xi}_i}\right)-\frac{\partial L}{\partial\xi_i}=F_i+\tau_i \tag{5}$$

where $F_i$ are the nonconservative forces, $\tau_i$ is the applied torque on the system, and $\vec{\xi}$ is the deflection vector.

$$[\xi_1,\xi_2,...,\xi_{n+1}]=[\theta,q_1(t),q_2(t),...,q_n(t)] \tag{6}$$

The kinetic energy of the system is then calculated:

$$T=\frac{1}{2}I_c\dot{\theta}^2+\frac{1}{2}\rho A\int_0^l\dot{\vec{R}}\dot{\vec{R}}\,dx+\frac{1}{2}m_s\dot{X}_B^2 \tag{7}$$

where $m_s$ is the mass of the slider, $\vec{X}_B$ is the velocity of the connecting rod end point, $I_c$ is the moment of inertia of the crank, and $\rho$, $A$ are the density and cross section of the connecting rod, respectively.

$$\dot{\vec{R}}\dot{\vec{R}}=\left(-r\dot{\theta}\sin\theta+\dot{w}\cos\psi+(x+w)\frac{d\cos\psi}{dt}\right)^2+\left(r\dot{\theta}\cos\theta+\dot{w}\sin\psi+(w-x)\frac{d\sin\psi}{dt}\right)^2 \tag{8}$$

$$\vec{X}_B=\left(-r\dot{\theta}\sin\theta+x\frac{d\cos\psi}{dt}\right)\vec{i} \tag{9}$$

The dependent coordinate $\psi$ is then omitted using the holonomic constraint of the slider–crank mechanism (Eq. (3)). The potential energy of the mechanism is given by

$$U=\frac{1}{2}\int_0^l EI\left(\frac{\partial^2w}{\partial x^2}\right)^2dx+m_cg\frac{r}{2}\sin\theta \tag{10}$$