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A fast hybrid primal heuristic for multiband robust capacitated network design with multiple time periods☆

Fabio D'Andreagiovanni[a,b,c,*], Jonatan Krolikowski[a], Jonad Pulaj[a]

[a] Department of Optimization, Zuse-Institut Berlin (ZIB), Takustr. 7, 14195 Berlin, Germany
[b] DFG Research Center MATHEON, Technical University Berlin, Straße des 17. Juni 135, 10623 Berlin, Germany
[c] Einstein Center for Mathematics Berlin (ECMath), Straße des 17. Juni 135, 10623 Berlin, Germany

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ABSTRACT

We investigate the Robust Multiperiod Network Design Problem, a generalization of the Capacitated Network Design Problem (CNDP) that, besides establishing flow routing and network capacity installation as in a canonical CNDP, also considers a planning horizon made up of multiple time periods and protection against fluctuations in traffic volumes. As a remedy against traffic volume uncertainty, we propose a Robust Optimization model based on Multiband Robustness (Büsing and D'Andreagiovanni, 2012), a refinement of classical Γ-Robustness by Bertsimas and Sim that uses a system of multiple deviation bands.

Since the resulting optimization problem may prove very challenging even for instances of moderate size solved by a state-of-the-art optimization solver, we propose a hybrid primal heuristic that combines a randomized fixing strategy inspired by ant colony optimization and an exact large neighbourhood search. Computational experiments on a set of realistic instances from the SNDlib show that our original heuristic can run fast and produce solutions of extremely high quality associated with low optimality gaps.

1. Introduction

In the last two decades, telecommunications have increasingly pervaded our everyday life and the volume of traffic sent and exchanged over networks has astonishingly increased: major companies like Nokia Siemens Networks expect that the increase in the amount of traffic will strongly continue, reaching a volume of more than 1000 exabyte per year in fixed networks by 2015 [34]. This dramatic growth that telecommunications have experienced has greatly compounded the challenge for network professionals, who are facing design problems of increasing complexity and difficulty. In order to cope with traffic growth, the professionals have to plan much in advance how the network will be expanded in topology and capacity to accommodate the new traffic. This is especially important in the case of fixed networks, which require (costly) digging operations for the installation of cables in areas with possibly high population density.

To make the design task even more complicated, the future behaviour of traffic over a network is not exactly known when the network is designed and thus the decision problem is also affected by tricky data uncertainty: until recent times data uncertainty has been generally neglected in real studies. However, as indicated by recent industrial cooperations between industry and academia, (e.g., [2,3,10,30]), professionals are not only becoming aware of the importance of adopting mathematical optimization to take better decisions, but are also understanding the necessity of considering data uncertainty, in order to avoid unpleasant surprises like infeasibility of implemented solutions due to data deviations.

The task of designing a telecommunication network essentially consists in establishing the topology of the network and the technological features (e.g., transmission capacity and rate) of its elements, namely nodes and links. One of the most studied problem in network design is the *Capacitated Network Design Problem (CNDP)*: the CNDP consists in minimizing the total installation cost of capacity modules in a network to route traffic flows generated by users. The

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* Corresponding author at: Department of Optimization, Zuse-Institut Berlin (ZIB), Takustr. 7, 14195 Berlin, Germany. Tel.: +49 3084185460.

E-mail addresses: d.andreagiovanni@zib.de (F. D'Andreagiovanni), krolikowski@zib.de (J. Krolikowski), pulaj@zib.de (J. Pulaj).

URL: http://www.dis.uniroma1.it/ fdag/ (F. D'Andreagiovanni).

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CNDP is a central problem in network optimization, which appears in many real-world applications. For an exhaustive introduction to it, we refer the reader to [1,5,8].

In this paper, we focus on the development of a new Robust Optimization model to tackle traffic uncertainty in the *Multiperiod Capacitated Network Design Problem (MP-CNDP)*. This problem constitutes a natural extension of the classical CNDP, where, instead of a single design period, we consider the design over a time horizon made up of multiple periods. Moreover, traffic uncertainty is taken into account to protect design solutions against deviations in the traffic input data, which may compromise feasibility and optimality of solutions.

We immediately stress that, though the problem of optimally designing networks over multiple time periods is not new and can be traced back at least to the seminal work by Christofides and Brooker [17], to the best of our knowledge, the MP-CNDP has received very little attention and just a few works have investigated it – essentially, [31,25]. Checking literature, several other works dealing with multiperiod design of networks can be found (just to make a couple of examples, [26,27]): however, all these works consider problems that are application-specific or are sensibly different from the more general setting that we consider here and we thus avoid a more detailed discussion of them ([26] studies capacity expansion problem in access networks with tree topology, whereas [27] considers the design of utility networks modeled by non-linear mathematical programs).

Our main references for this work, namely [31,25], point out the difficulty of solving multiperiod CNDP problems already for just two periods and even in (easier) contexts: [31] considers CNDP when traffic flows may be split, whereas [25] considers a pure routing problem in satellite communications. Our direct and more recent computational experience have confirmed the challenging nature of the MP-CNDP, even for instances of moderate size with a low number of time periods and solved by a state-of-the-art optimization solver.

Uncertain versions of the MP-CNDP where traffic uncertainty is considered have also been neglected, even though especially in the last years there has been an increasing interest in network design under traffic uncertainty for the single design period case (e.g., [2,3,30]).

In this work, our main original contributions are:

1 the first Robust Optimization model for tackling traffic uncertainty in Multiperiod CNDP. Specifically, we adopt Multiband Robustness, a new model for Robust Optimization recently introduced by Büsing and D'Andreagiovanni [14];
2 a hybrid primal heuristic, based on the combination of a randomized rounding heuristic resembling *ant colony optimization* [22] with an exact large neighborhood search called RINS [19]. We stress here that our aim was not to use a standard implementation of an ant colony algorithm: we wanted instead to strengthen the performance of the ant algorithm using highly valuable information from linear relaxations of the considered optimization problems. Using this information allowed us to define a very strong ant construction phase, which produces very high quality solutions already before the execution of any local search;
3 analytically proving how to solve the linear relaxation of the Multiperiod CNDP in closed form, thus obtaining a substantial reduction in solution times w.r.t. our first algorithm presented in [20];
4 computational experiments over a set of realistic instances derived from the Survivable Network Design Library (SNDlib) [33], showing that our hybrid algorithm is able to produce solutions of extremely high quality associated with very small optimality gap.

The remainder of this paper is organized as follows: in Section 2, we review a canonical model for the CNDP; in Section 3, we introduce the Multiperiod CNDP and we study its linear relaxation; in Section 4, we introduce the new formulation for Robust Multiperiod CNDP; in Sections 5 and 6, we present our hybrid heuristic and computational results.

2. The Capacitated Network Design Problem

The CNDP is a central and highly studied problem in Network Optimization that appears in a wide variety of real-world applications (see [1,5] for an exhaustive introduction to it) and can be essentially described as follows: given a network and a set of demands whose flows must be routed between vertices of the network, we want to install capacities on network edges and route the flows through the network, so that the capacity constraint of each edge is respected and the total cost of installing capacity is minimized. More formally, we can characterize the CNDP through the following definition.

Definition 1 *(The Capacitated Network Design Problem – CNDP).* Given

- a network represented by a graph $G(V, E)$, where V is the set of vertices and E the set of edges,
- a set of commodities C, each associated with a traffic flow d_c to route from an origin s_c to a destination t_c,
- a set of admissible paths P_c for routing the flow of each commodity c from s_c to t_c,
- a cost γ_e for installing one module of capacity $\phi > 0$ on edge $e \in E$,

the CNDP consists in establishing the number of capacity modules installed on each edge $e \in E$ such that the resulting capacity installation has minimum cost and supports a feasible routing of the commodities. A feasible routing assigns each commodity $c \in C$ to exactly one feasible path $p \in P_c$. □

Referring to the notation introduced above and introducing the following two families of decision variables:

- Binary *path assignment variables* $x_{cp} \in \{0, 1\}\ \forall\, c \in C,\ p \in P_c$ such that:

$$x_{cp} = \begin{cases} 1 & \text{if the entire traffic of commodity } c \text{ is routed on path } p \\ 0 & \text{otherwise,} \end{cases}$$

- Integer *capacity variables* $y_e \in \mathbb{Z}_+,\ \forall\, e \in E$, representing the number of capacity modules installed on edge e,

we can model the CNDP as the following *integer linear program*:

$$\min \sum_{e \in E} \gamma_e\, y_e \qquad \text{(CNDP – IP)}$$

$$\sum_{c \in C} \sum_{p \in P_c:\, e \in p} d_c\, x_{cp} \le \phi\, y_e \qquad e \in E \tag{1}$$

$$\begin{aligned} &\sum_{p \in P_c} x_{cp} = 1 && c \in C \\ &x_{cp} \in \{0, 1\} && c \in C, p \in P_c \\ &y_e \in \mathbb{Z}_+ && e \in E\,. \end{aligned} \tag{2}$$

The objective function minimizes the total cost of capacity installation. Capacity constraints (1) impose that the summation of all flows routed through an edge $e \in E$ must not exceed the capacity installed on e (equal to the number of installed modules represented by y_e multiplied by the capacity ϕ granted by a single