Applied Soft Computing 22 (2014) 518–527

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Practical computing with interactive fuzzy variables

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ARTICLE INFO

Article history:
Received in revised form 4 November 2013
Available online 28 April 2014

Keywords:
Fuzzy Calculator
Alpha-cuts
Interactivity
Triangular norm
Possibilistic clustering

ABSTRACT

The importance of epistemic uncertainty in engineering is being recognized more and more. In this work, epistemic uncertainty is captured through the use of fuzzy variables, i.e. variables that are described in terms of possibility distributions. Taking into account epistemic uncertainty implies that one should also be able to propagate such uncertainty through increasingly complex models. In earlier work, we have developed a general tool, called Fuzzy Calculator, to propagate epistemic uncertainty regarding non-interactive fuzzy input variables. In reality, however, the fuzzy input variables of a model are often interactive. Taking into account this interactivity reduces the uncertainty on the fuzzy output variable and should therefore always be aimed for. However, the full determination of the joint possibility distribution of the fuzzy input variables is not an easy task. In literature, it is often claimed that triangular norms can be used to model interactivity between fuzzy variables. Making use of a generalization of Nguyen's alpha-cut approach, we have developed a general-purpose Fuzzy Calculator that is able to take into account interactivity modelled by a triangular norm. However, the absence of a general strategy to select an appropriate triangular norm often results in the use of other strategies to model interactivity in practice. Therefore, we have also applied our Fuzzy Calculator to a case study where the interactivity between the fuzzy input variables is identified through possibilistic clustering. The results illustrate that this Fuzzy Calculator is an efficient tool to propagate epistemic uncertainty regarding non-interactive as well as interactive fuzzy input variables.

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1. Introduction

Zimmerman [54] describes uncertainty as "the lack of necessary information to quantitatively and qualitatively ... describe, prescribe or predict deterministically and numerically a system, its behaviour or other characteristics". Taking into account uncertainty in mathematical modelling is a subject of great interest to many engineering applications (see among others [3,27,31,48,49]). Uncertainty can be categorized into two groups, namely aleatoric and epistemic uncertainty [1,21,30]. Aleatoric or stochastic uncertainty results from the natural randomness in certain processes. It describes an inherent variability that is present in the variables of interest and that cannot be reduced by enhancing the available knowledge. This kind of uncertainty can be described by probability theory. Epistemic or systematic uncertainty stems from incomplete knowledge about the variables or parameters of interest. This type of uncertainty is related to the state of knowledge: while the variables have a unique value in reality, this value is not known exactly, either due to experimental limitations or due to approximations in the model. Even the most complex model of a real system necessarily involves a series of assumptions and approximations, which are required to compensate for our incomplete understanding of the real world and the complexity of its description. Consequently, the variables in such an approximate model do not always directly correspond to real physical variables and their value is therefore uncertain. In this paper, we restrict our attention to epistemic uncertainty. Fuzzy set theory has been developed to incorporate epistemic uncertainty into mathematical models [52]. Variables subject to this type of uncertainty are represented as fuzzy variables. If these variables act as the inputs to a model, the uncertainty should be propagated through the model using Zadeh's extension principle [53], which extends functions of real numbers to functions with fuzzy quantities as arguments.

However, the application of the extension principle to compute with fuzzy quantities is a complex matter. Nguyen [40] developed a more practical approach based on alpha-cuts, which is applicable to continuous functions and upper semi-continuous fuzzy quantities with compact support, describing non-interactive fuzzy input variables. This approach transforms the problem into an optimization problem. By optimizing the function on a number of alpha-cuts,

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http://dx.doi.org/10.1016/j.asoc.2014.04.003
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it is possible to construct the possibility distribution of the fuzzy quantity describing the output variable. In the literature, several methods based on this transformation are available for (locally) monotone functions [13,14,17,22,38,43,45] where the function reaches its extreme values in some of the vertices of the hyperrectangle generated by the alpha-cuts of the possibility distributions of the non-interactive fuzzy input variables. For non-monotone functions, Scheerlinck et al. [44] developed a Fuzzy Calculator, which applies Particle Swarm Optimization (PSO) to optimize the function over the entire hyperrectangle. It is thus a general tool to determine the possibility distribution of the fuzzy output variable when propagating non-interactive fuzzy input variables through a function or model. However, in many applications the fuzzy input variables of a function or model are interactive, for example when they are associated with the same physical process or property. By taking the interactivity into account, which corresponds to additional knowledge about the joint state of the variables, the uncertainty on the output variable can be reduced. However, it is often complex and practically infeasible to know and describe the exact interactivity pattern for a set of fuzzy variables. De Cooman [11] illustrated that triangular norms can be used to describe interactivity between fuzzy variables. However, there is still a lot of research needed in this area and at this moment there is no general strategy to determine which triangular norm should be used. Therefore, other methods can be applied to describe the interactivity between the fuzzy variables, for example by applying a possibilistic clustering algorithm to joint measurements of the relevant variables [51]. Note that interactivity is not used here to simply refer to the reoccurrence of certain variables in non-linear functions or models, as is the case in for instance Gladysz and Kasperski [26]. The propagation of uncertainty through such non-linear functions can already be done using the Fuzzy Calculator developed in Scheerlinck et al. [44]. To the authors' knowledge, no general approach exists in the literature to propagate uncertainty in the presence of true interactivity between non-identical fuzzy input variables of the model.

Fullér and Keresztfalvi [23] generalized Nguyen's theorem for interactive fuzzy input variables. Based on this generalization, the current paper presents a modified Fuzzy Calculator that allows to take interactivity into account. It is therefore referred to as the 'generalized Fuzzy Calculator'. Firstly, the generalized Fuzzy Calculator is applied to the same test functions as described in Scheerlinck et al. [44]. Some basic triangular norms will be used to describe the interactivity between the fuzzy input variables of these test functions. In this way, we can evaluate the performance of the generalized Fuzzy Calculator.

Next, the generalized Fuzzy Calculator is applied to a case study in which we want to determine the possibility distribution of the output of a physically-based surface backscattering model. Such models are developed to describe the relation between the backscatter coefficients of microwaves, obtained in radar data, the roughness parameters of the surface, the surface soil moisture content, vegetation characteristics, the frequency polarization and incidence angle. The surface soil moisture content is an important property in the understanding and modelling of meteorology, hydrology and agriculture [12,46]. Soil moisture is responsible for the partitioning of precipitation in surface runoff and infiltration, as well as the partitioning of the incoming radiation in latent and sensible heat fluxes [19,29]. Therefore, monitoring soil moisture is of high importance. However, as field measurements of soil moisture are not feasible for large areas, there is a need for alternative methods to retrieve information about the moisture content of the soil's surface. A solution to this problem is provided by the relation that exists between the backscatter coefficients of microwaves, vegetation, the roughness parameters of the surface and the surface soil moisture. However, in order to be useful for hydrological applications, the backscatter coefficient needs to be converted into the corresponding soil moisture content. Therefore, we can rely on physically-based surface scattering models. In particular, this case study focuses on the Integral Equation Model (IEM) [24,25] that is applicable for bare soils (*i.e.* no significant vegetation is present). By inverting this model, we can determine the soil moisture content based on the roughness parameters and the backscatter coefficient. Since the roughness parameters offer only an approximate and statistical description of the true structure of the surface, they are difficult to measure directly and should thus be described as interactive fuzzy variables [6,9,37,42,50,51]. In order to determine the interactivity between the two roughness parameters, we follow the approach in Vernieuwe et al. [51] and apply a possibilistic clustering algorithm. In this work, the extension principle was also used in order to propagate the uncertain roughness parameters through the inverted IEM. However, in contrast to the Fuzzy Calculator, only the boundary of the search space, corresponding to the interactive roughness parameters, was searched through. In the present paper uncertainty captured through the use of interactive fuzzy variables will be propagated through the model using the generalized Fuzzy Calculator; the resulting output is then compared to the results in Vernieuwe et al. [51].

This paper is set out as follows. Section 2 describes the difference between non-interactive and interactive fuzzy variables. Section 3 recapitulates the extension principle generalized for interactive fuzzy variables. Section 4 describes the modifications to the Fuzzy Calculator. Section 5 discusses the results of the generalized Fuzzy Calculator applied to some test functions. Section 6 introduces the case study to which the generalized Fuzzy Calculator is applied. Finally, Section 7 contains our conclusions.

2. Non-interactive versus interactive fuzzy variables

When studying the propagation of epistemic uncertainty through a model with n fuzzy input variables X_i $(i = 1, \ldots, n)$, it is not always valid to assume that these variables are not interactive. In general, interactivity between the variables is present and a full description requires detailed knowledge of the joint possibility distribution $\pi_{X_1,\ldots,X_n}$. Non-interactivity corresponds to the special case where $\pi_{X_1,\ldots,X_n}(x_1, \ldots, x_n) = \min(\pi_{X_1}(x_1), \ldots, \pi_{X_n}(x_n))$, with π_{X_i} $(i = 1, \ldots, n)$ the marginal possibility distributions of the fuzzy input variables. In a non-fuzzy setting where the possibility distributions π_{X_i} represent (crisp) sets A_i (*i.e.* $\pi_{X_i}(x_i) = 1$ if $x_i \in A_i$, $\pi_{X_i}(x_i) = 0$ elsewhere), non-interactivity means that none of the combinations $(x_1, \ldots, x_n) \in A_1 \times \cdots \times A_n$ is deemed impossible.

In case of interactivity, the joint possibility distribution is a complex object to describe or determine. One convenient way to simplify the description is by modelling the joint possibility distribution as $\pi_{X_1,\ldots,X_n}(x_1, \ldots, x_n) = T(\pi_{X_1}(x_1), \ldots, \pi_{X_n}(x_n))$, where the function T replaces the minimum operator of the non-interactive case and transforms the individual possibility degrees $\pi_{X_1}(x_1)$ to $\pi_{X_n}(x_n)$ into an appropriate joint possibility degree. Such functions T are called triangular norms. The purpose of triangular norms (shortly t-norms) is to generalize the intersection of classical sets to fuzzy sets or the Boolean conjunction of classical logic to fuzzy logic. A t-norm is a binary operation T on the unit interval [0, 1], *i.e.* a function $T : [0, 1]^2 \to [0, 1]$ with the following properties for all a, b, $c \in [0, 1]$ [34,35]:

$$
\begin{aligned}
&\text{(T1)} \quad T(a, b) = T(b, a) && \text{(commutativity)}\\
&\text{(T2)} \quad T(a, T(b, c)) = T(T(a, b), c) && \text{(associativity)}\\
&\text{(T3)} \quad T(a, b) \leq T(a, c) \quad \text{whenever} \quad b \leq c && \text{(monotonicity)}\\
&\text{(T4)} \quad T(a, 1) = a && \text{(boundarycondition)}
\end{aligned}
\tag{1}
$$

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