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New exact traveling wave solutions of the (4+1)-dimensional Fokas equation

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ABSTRACT

In this paper, two distinct methods are applied to look for exact traveling wave solutions of the (4+1)-dimensional nonlinear Fokas equation, namely the modified simple equation method (MSEM) and the extended simplest equation method (ESEM). Some new exact traveling wave solutions involving some parameters are obtained. The solitary wave solutions can be extracted by assigning special values of these parameters. The obtained solutions show the simplicity and efficiency of the used approaches that can be applied for nonlinear equations as well as linear ones.

1. Introduction

Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in many fields of science such as applied mathematics, engineering, chemistry, biology, physics, and many others. Therefore, extracting traveling wave solutions of these equations becomes an increasingly important subject in nonlinear sciences. In the past several decades, much attention has been paid for this purpose and various powerful methods have been proposed by a lot of researchers such as Hirota's bilinear transformation method [1,2], the Exp-function method [3], the $\exp(-\Phi(\xi))$-expansion method [4], the (G'/G)-expansion method [5–11], the Adomian decomposition method [12,13], the sine–cosine method [14], the homotopy perturbation method [15], the variational iteration method [13,16,17], the F-expansion method [18,19], the first integral method [20–23], the reduced differential transform method [24,25], the Jacobi elliptic function method [26], the modified simple equation method [27–30], the extended simplest equation method [31–35] and so on. The potential errors in finding the traveling wave solutions of nonlinear differential equations have been handled by Kudryashov [36]. Nevertheless, it is worthwhile to mention that there is no general unified method that can be used to obtain analytical solutions for all types of NPDEs.

Consider the (4+1)-dimensional nonlinear Fokas equation

$$4u_{tx} - u_{xxxy} + u_{xyyy} + 12u_x u_y + 12uu_{xy} - 6u_{zw} = 0. \tag{1}$$

This equation has been recently derived by Fokas [37] by extending the Lax pairs of the integrable Kadomtsev–Petviashvili (KP) and Davey–Stewartson (DS) equations to some higher-dimensional nonlinear wave equations.

The KP and DS equations have been proposed in nonlinear wave theory to describe the surface waves and internal waves in straits or channels of varying depth and width [38] and the evolution of a three-dimensional wave-packet on water of

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finite depth [39], respectively. Therefore, the physics of the Fokas equation (1) follows necessarily from the physical nature of the KP and DS equations.

The important applications of the (4+1)-dimensional nonlinear Fokas equation lie in real world problems considered as a higher dimensional integrable model in mathematical physics.

The importance of the Fokas equation (1) suggests that the idea of complexifying time can be investigated in the context of modern field theories via the existence of integrable nonlinear equations in four spatial dimensions involving complex time [37]. Therefore, it is necessary to investigate the analytic solutions of the (4+1)-dimensional Fokas equation (1).

More recently, (4+1)-dimensional Fokas equation have been investigated by several authors. Yang and Yan [40] investigated symmetries of Eq. (1) including Lie point symmetries as well as the potential symmetries. Lee et al. [41] obtained many traveling wave solutions of Eq. (1) by using modified tanh–coth method, extended Jacobi elliptic function method and the Exp-function method. Kim and Sakthivel [42] used the(G'/G)-expansion method to obtain exact solutions of Eq. (1). He [43] studied Eq. (1) by using the extended F-expansion method to find some new exact solutions. Zhang et al. [44] employed Hirota's bilinear method to solve Eq. (1).

The aim of this article is to conduct the analysis on Eq. (1) by utilizing the modified simple equation method (MSEM) and the extended simplest equation method (ESEM) to seek the exact traveling wave solutions of the (4+1)-dimensional Fokas equation which is a recent higher-dimensional nonlinear wave equation appears in mathematical physics.

The article is prepared as follows: In Section 2, a brief description of the MSEM and the ESEM is given. In Section 3, we present new applications of the proposed methods to solve the (4+1)-dimensional Fokas equation. In Section 4, some discussion and conclusions are given.

2. Methodology

In this section, we exhibit the methodology of the modified simple equation method (MSEM) and the extended simplest equation method (ESEM).

Consider a general nonlinear PDE, say in five independent variables x, y, z, w and t, in the form

$$P\left(u, u_t, u_x, u_y, u_z, u_w, u_{tt}, u_{xx}, u_{yy}, u_{zz}, u_{ww}, \ldots\right) = 0, \tag{2}$$

where P is a polynomial of $u(x, y, z, w, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are included.

Using the traveling wave transformation

$$u(x,t) = u(\xi), \xi = \alpha x + \beta y + \chi z + \rho w + \varepsilon t + \xi_0, \tag{3}$$

where ε is the wave speed and ξ_0 is an arbitrary constant, to transform Eq. (2) to a nonlinear ordinary differential equation (ODE) as

$$Q\left(u, \varepsilon u', \alpha u', \beta u', \chi u', \rho u', \varepsilon^2 u'', \alpha^2 u'', \beta^2 u'', \chi^2 u'', \rho^2 u'', \ldots\right) = 0 \tag{4}$$

where the prime denotes the derivation with respect to ξ.

2.1. Description of the modified simple equation method

The foremost steps of MSEM can be outlined as follows [27–30]:

Step 1: Suppose that the solution of Eq. (4) can be expressed by a finite series of the form

$$u(\xi) = \sum_{i=0}^{N} a_i \left(\frac{\phi'(\xi)}{\phi(\xi)}\right)^i \tag{5}$$

where a_i $(i = 0, 1, 2, \ldots, N)$ are arbitrary constants to be determined, such that $a_N \neq 0$, and $\phi(\xi)$ is an unidentified function to be determined subsequently.

Step 2: The positive integer N can be determined by using the homogeneous balance technique between the highest order derivatives and the nonlinear terms come out in Eq. (4).

Step 3: We calculate all the necessary derivatives $u', u'', \ldots$ and substitute Eq. (5) into Eq. (4). As a result of this substitution, we obtain a polynomial of $\phi^{-j}(\xi)$ with the derivatives of $\phi(\xi)$. We equate all the coefficients of $\phi^{-j}(\xi)$ to zero, where $j \geq 0$. This procedure yields a system of equations which can be solved to find a_i $(i = 0, 1, 2, \ldots, N)$, $\phi(\xi)$ and $\phi'(\xi)$.

Step 4: We substitute the values of a_i, $\phi(\xi)$ and $\phi'(\xi)$ into Eq. (5) to complete the determination of exact solutions of Eq. (2).

2.2. Description of the extended simplest equation method

The ESEM can be described in the following steps [31–35]:

Step 1: For the simplest equation method, suppose the solution of Eq. (4) can be expressed as in the form

$$u(\xi) = \sum_{i=0}^{N} a_i (\phi(\xi))^i, \tag{6}$$