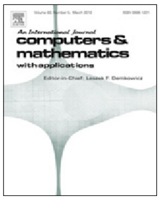
Computers and Mathematics with Applications 74 (2017) 229–248

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa



Numerical analysis of coupled heat and moisture transport in masonry

Tomáš Krejčí*, Jaroslav Kruis, Michal Šejnoha, Tomáš Koudelka

Faculty of Civil Engineering, Czech Technical University in Prague, Czech Republic

ARTICLE INFO

Article history:
Available online 5 April 2017

Keywords:
Heat and moisture transfer
Masonry
Finite element method
Parallel computing
Homogenization
Processor farm

ABSTRACT

Coupled analysis of heat and moisture transport in real world masonry structures deserves a special attention because the spatial discretization by the finite element method leads usually to large number of degrees of freedom. Thin mortar layers and large bricks or stones have very different material properties and the finite element mesh has to be able to describe correct temperature and moisture fields in mortar and in its vicinity in the blocks. This paper describes two possible solutions of such problems. The first solution is based on the domain decomposition method executed on parallel computers, where the Schur complement method is used with respect to non-symmetric systems of algebraic equations. The second alternative method is the application of a multi-scale approach in connection with a processor farming method, where the whole structure is described by a reasonably coarse finite element mesh, called the macro-scale problem, and the material parameters are obtained from the lower-level problems, called the meso-scale problem, by a homogenization procedure. In this procedure, the macro-problem is assigned to the master processor and the meso-scale problems belong to the slave processors in the processor farm.

1. Introduction

Historical masonry structures such as, e.g., masonry arch bridges are seriously influenced by climatic loading. In many cases, it is responsible for the nucleation and further development of cracks and damage. Restoration and reconstruction of such damaged structures need complex analyses which are able to combine experimental work and numerical simulation [1]. A thermo-hygro-mechanical simulation of the response of masonry structures to climatic loadings can be shown as a suitable example of such complex numerical analysis. Numerical modelling of coupled hygro-thermal problems in masonry by the finite element method is hardly solvable on single processor computers. The huge number of degrees of freedom is usually obtained in case of detailed finite element discretization of thin mortar layers between bricks and stones. Miehe and Bayreuther [2] suggest to solve such systems by multigrid method but very large systems cannot be executed on a single processor computers. One of the possible solutions is an application of a domain decomposition method [3–5] in connection with parallel computers. There are two groups of domain decomposition methods: non-overlapping and overlapping methods. In the case of overlapping method, suitable overlap is prescribed between two adjacent subdomains. Overlapping methods are represented by the Schwarz method. In non-overlapping methods, subdomains share only nodes, edges or surfaces. The Schur complement method is very robust tool for solution of symmetric as well as non-symmetric systems. Another example of non-overlapping method is FETI (finite element tearing and interconnecting) method. Since 1991 when the method

---

* Corresponding author.
E-mail address: krejci@cml.fsv.cvut.cz (T. Krejčí).

http://dx.doi.org/10.1016/j.camwa.2017.03.022

was introduced [6], several variants were formulated. The original method is denoted FETI-1. In 1998, so called FETI-2 was introduced in [7,8] and FETI-DP was published in [9]. FETI method with all floating subdomains was described in [10].

The other possible solution of the given difficulties of masonry and heterogeneous materials and structures can be seen in an application of parallel computing and a multi-scale approach. When dealing with these problems, the application of homogenization techniques is inevitable [11]. Solving a set of problem equations on a meso-scale (a composition of stone blocks and mortar) provides with up-scaled macroscopic equations. They include a number of effective (macroscopic) transport parameters, which are necessary for a detailed analysis of the state of a structure as a whole [12]. Such a multi-scale approach to coupled heat and moisture transfer is based on a clear description of transport phenomena. An extensive review of this topic can be found in [13]. Averaging theories (a micromechanics-based approach), formulated, e. g., in [14,15], are considered as a counterpart to phenomenological models (a macromechanics-based approach), see [16,17]. When calculating the heat and moisture transfer in building materials, phenomenological models are still preferred to averaging ones. Both approaches are explained in detail in [18]. A phenomenological model proposed by Künzel and Kiessl [19] was chosen for studying transport processes in masonry structures due to its ability to describe all substantial phenomena of heat and moisture transport.

While models for transport processes have been developed during several decades, the computational methods for multi-scale modelling of these processes in masonry on meso and macro scales have emerged only recently. An original approach to homogenization of transient heat transfer for some composite materials is proposed in [20]. The complex multi-scale analysis for pure heat transfer in heterogeneous solids is offered in [11], where the authors established a macro to micro transition in terms of the applied boundary conditions and likewise a micro to macro transition formulated in the form of consistent averaging relations. The similar study with applications to textile composites can be found, e.g. in [21]. Homogenization strategies for coupled heat and mass transfer used in this paper are described in detail in [22,12,23]. Therein, governing equations of the coupled heat and moisture transport are derived in the framework of coupled two-scale analysis (the first-order homogenization approach) in conjunction with the finite element method. The homogenized macro-scale fields are found from the solution of a certain sub-scale (meso-scale) problem performed on a representative volume element (RVE).

This paper presents two computational methods based on parallel computing for effective simulation of coupled heat and moisture transfer in masonry. The first approach is the domain decomposition method, where the original domain is split into several smaller subdomains which are assigned to processors of a parallel computer. The second approach is the processor farming method in connection with a multi-scale analysis. In this method, each macro-scopic integration point or each finite element is connected with a certain meso-scopic problem represented by an appropriate representative volume element or a periodic unit cell (PUC). The solution of a meso-scale problem then provides effective parameters needed on the macro-scale. Such an analysis is suitable for parallel computing because the meso-scale problems can be distributed among many processors. In this regard, the master–slave strategy can be efficiently exploited. The processor farming method based on multi-scale analysis differs from classical parallel computing methods which come out from the domain decomposition. The macro-problem is assigned to the master processor while the solution at the meso-level is carried out on slave processors. At each time step the current temperature and moisture together with the increments of their gradients at a given macro-scopic integration point are passed to the slave processor (imposed onto the representative volume element), which, upon completing the small scale analysis, sends the homogenized data (effective conductivities, averaged storage terms and fluxes) back to the master processor. It means that the conductivity and capacity matrix of an element of the macro-scale problem are assembled as a result of the homogenization on meso-scale.

The presented parallel methods have been implemented into a parallel version of SIFEL (Simple Finite Elements) code [24] with the distributed memory scheme and the MPI communication library. The code uses the master and slaves concept, where the master processor manages communication among all processors and it also controls the computation.

The paper consists of seven sections. After the first Introduction, the basic approach based on Künzel and Kiessl coupled heat and moisture transfer model is summarized in Section 2. The basic principles of domain decomposition together with Schur complement method are discussed in Section 3. The first order homogenization procedure is described in Section 4. Section 5 deals with the processor farming method and results of two numerical simulations of heat and moisture transfer are illustrated in Section 6. The first benchmark, the 2D analysis of a brick wall, shows the application of both methods presented. The second example is the analysis of Charles bridge in Prague which illustrates the application of processor farming method to a real world masonry structure. Finally, the benefits of used methods and their comparison are discussed in Section 7.

Customary matrix notation is used throughout the text. Matrices are denoted by uppercase boldface italic letters, e.g., $\boldsymbol{D}$, $\boldsymbol{P}$, etc. Conversely, lowercase boldface italic letters stand for vectors $\boldsymbol{g}$, $\boldsymbol{j}$, etc. $\nabla\cdot$ is the divergence operator, while ∇ stands for the gradient operator.

2. Description of coupled heat and moisture transfer and numerical solution

For the description of transport phenomena, phenomenological modelling has still been preferred to micromechanics-based (averaging) description outlined, e.g., in [18]. In this paper, we focus just on one particular model implemented in the SIFEL computer code by which all the results discussed in this paper were obtained. It is the diffusion model proposed by Künzel and Kiessl [19]. It is much simpler than micromechanics-based models and it describes all substantial phenomena very well. This phenomenological model introduces two unknowns, relative humidity φ [-] and temperature T [K] in a material point. The practical usage of the main advantages of this model can be seen, e.g., in Refs. [25,26], where building